Anti-sway fixed-order control of bridge cranes with varying rope length

Alberto L. Cologni, Michele Ermidoro, Simone Formentin, *Member, IEEE* and Fabio Previdi, *Member, IEEE*

***Abstract*— In this paper, a gain-scheduling control law is proposed to attenuate the oscillations in overhead cranes induced by manual operations, for different values of the rope length. To take into account the practical limits in controller implementation, a fixed-order controller is tuned, by enforcing certain robustness and performance constraints. The proposed strategy is experimentally tested on a real bridge crane and compared to a time-invariant solution.**

I. INTRODUCTION

It is well known that overhead cranes suffer from safety problems due to the flexibility of the rope linking the load to the hoist. In fact, the load swinging is usually very poorly damped and the uncontrolled sway might be dangerous for human operators. Moreover, the oscillations require a certain time to stop, thus slowing the overall movement time.

Many approaches have been proposed to solve the problem of the load oscillations induced by the movement of the crane. For instance, a second order sliding mode control has been used in [1] while in [2] an adaptive sliding mode control is employed. The approaches in [3] and [4] adopt a time optimal perspective, while [5] and [6] propose an open-loop input shaping method.

All the above solutions do not consider the fact that the rope length and the mass of the load may change during the system operation; nevertheless, such events occur quite often in practical working cycles. For this reason, gain-scheduled controllers appear to be a suited solution to the problem of sway suppression.

Among the solutions addressing the problem at hand from a gain-scheduling perspective, the method in [7] considers the length of the rope as a scheduling signal for an implicit gain scheduling controller and employs the knowledge of the upper bounds in the rate of change of such a parameter to ensure the stability of the closed-loop system. In [8] a state-space interpolation method is used for an analogous design purpose. This method, albeit providing good performance, does not ensure the stability of the systems in case of parameter variations.

As far as we are aware, in all the above contributions, the simplicity of the controller structure and the robustness to model uncertainty are not explicit requirements.

In this paper, the problem of sway cancellation in overhead bridge cranes is tackled in a gain-scheduled rationale, but also taking into account the simplicity of the final controller (to make it suitable for implementation on a wide range of micro-controllers or PLCs) and finding the best trade off between performance and robustness. More specifically, a fixed-order gain scheduling controller is designed (thus with a user-defined structure) aimed to minimize the integral error but also constraining the main robustness margins. The proposed method has been first introduced in [9], but herein the minimization of the settling time is also considered.

The fixed-order gain scheduling controller is experimentally implemented on a real bridge crane and the achieved performance is compared to that of a linear time-invariant controller tuned according to the same specifications. The experiments show that, although the employed structure is very simple, the gain scheduling controller is able to suppress the sway in all the conditions of interest, unlike the time-invariant solution. Finally it has to be stressed that, in the proposed closed-loop solution, where the oscillations are estimated through proper measurements and automatically compensated by the feedback controller, the operator can still manually operate the system, without defining a-priori any reference trajectory.

The remainder of the paper is organized as follows. Section II describes the experimental setup and the problem statement. In Section III, the fixed-order gain-scheduling control design method is described, with a focus on how to select the different tuning knobs. Section IV presents the experimental results. The paper is ended by some concluding remarks.

A. L. Cologni, M. Ermidoro and F. Previdi are with the Control systems and Automation Laboratory, Università degli Studi di Bergamo, 24044 Dalmine (BG), Italy. Email to: michele.ermidoro@unibg.it

S. Formentin is with the Department of Electronics, Information and Bioengineering, Politecnico di Milano, 21033 Milano, Italy.

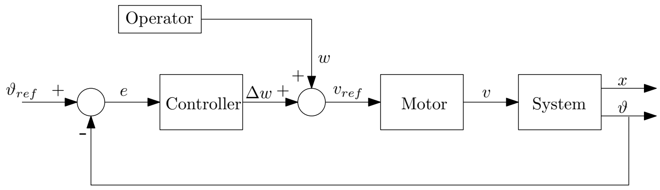

Fig. 1. Block diagram of the overall system.

II. PROBLEM STATEMENT AND EXPERIMENTAL LAYOUT

A description of the bridge crane, for each axis, is depicted in Figure 1. The operator, using a button panel, sends commands to the motors and varies the position x and the sway angle ϑ. The oscillation is then controlled by means of a feedback loop.

The purpose of the paper is to design a controller which is able to remove the sway without affecting the human/system interaction.

The bridge crane that will be used in this paper has a maximum payload of 20000 kg. On the X-axis and Y-axis,

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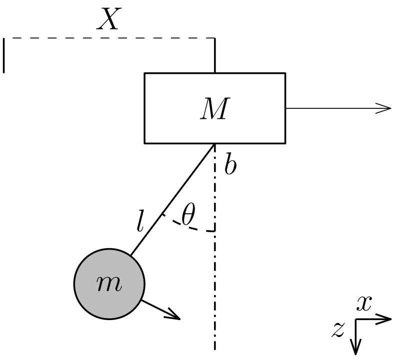

Fig. 2. Structure of a mono-dimensional bridge crane.

it can move at a maximum speed of 1 m/s, while on the Z-axis, it can lift the objects at 0.2 m/s. The bridge has an elevation from the ground of 7 m, while the trolley can span for 20 m while the bridge on the Y-axis can move for 80 m.

In order to estimate the oscillation angle, an inertial sendor composed by a tri-axial accelerometer and a tri-axial gyroscope has been placed on the rope that connects the load to the trolley. The angle is then estimated using the Extended Kalman filter described in ([10]).

The estimated angle is then measureed by a Programmable Logic Controller (PLC). The actuation chain, from the PLC command to the speed of the motor is not ideal: the bandwidth of the motor controller can be considered wide enough for our purposes, but the pure input/output delay cannot be neglected. This nominal delay has a fixed amount that can be a-priori identified and it is due to the disabling of the brakes.

III. Fixed-order controller design

The system is assumed to be completely decoupled as discussed in [11], so the model is built for a mono-axial cart-pendulum as the one visible in Figure 2.

To derive a control-oriented model of the system, some assumptions are made, as follows.

- The payload is connected to the trolley by a massless, rigid rope.
- The trolley and the bridge move along the track without slipping.
- The speed control system is assumed to be ideal, that is the actual speed is assumed to be equal to the reference one.
- The moment of inertia of the load is neglected, and it is treated as a point mass (notice that this approximation is valid also in case of a multi-wire rope [12]).

The model, built using the Eulero-Lagrange equations of motion, can be linearized about $\dot{\theta} = 0$, $\theta = 0$ and $u = 0$, obtaining, after simple elaborations [4]:

$$\frac{\theta(s)}{\dot{X}(s)} = F(s) = \frac{-\frac{1}{l}s}{s^2 + \frac{b}{ml^2}s + \frac{g}{l}} \tag{1}$$

The above model shows that, even for fixed load mass, a change of the rope length may heavily influence the system dynamics. For this reason, a time invariant controller may have poor performance and even encounter stability problems.

A. Gain-scheduling control

The procedure used to design the controller is based on the methodology described in [13]; in order to tune the fixed-order linearly parameterized gain-scheduled controller, a linear programming approach is used. The Nyquist diagram of the open-loop transfer function is shaped in order to respect some constraints which will guarantee lower bounds on the robustness margin and optimal closed loop load disturbance rejection in terms of Integrated Error (IE):

$$IE = \int_0^{\infty} |e(t)| dt \tag{2}$$

where $e(t)$ is the difference between the desired output and the measured output. The closed-loop stability is locally ensured (i.e., for fixed values of the scheduling parameter).

Once the structure of the controller and the constraints on the robustness and performance are defined, the problem is solved using an optimization algorithm. In particular, a linear programming problem is solved ([14]).

1) Plant Model: The method can be applied only to a particular class of SISO LPV systems: the plant model depends on a n_l-dimensional vector l of scheduling parameters and must have no Right Half-Plane (RHP) poles.

The definition of the n_l-dimensional vector will define a set of models. Suppose that this set covers all the range of values that can be assumed by the scheduling parameter and that it is available a sufficient amount of frequency points N to capture the dynamics of the systems; then, the plant model can be parameterized as

$$\mathcal{M} = \{\mathcal{F}(j\omega_k, l_i) \mid k = 1, \ldots, N;\ i = 1, \ldots, m\} \tag{3}$$

where ω_k is the vector of frequency in which the system will be evaluated and l_i is the vector of the scheduling parameter.

2) Controller definition: Consider the following class of controllers:

$$\mathcal{K}(s, l) = \rho^T(l)\,\phi(s) \tag{4}$$

with

$$\begin{aligned} \rho^T(l) &= \left[\rho_1(l), \rho_2(l), \ldots, \rho_{n_p}(l)\right] \\ \phi^T(l) &= \left[\phi_1(l), \phi_2(l), \ldots, \phi_{n_p}(l)\right] \end{aligned} \tag{5}$$

where n_p is the number of parameter ρ polinomially dependent from l and $\phi_i(s)$, $i = 1, \ldots,\ n_p$ are rational basis functions with no RHP poles. The dependence of ρ_i on the parameter l can be represented using a polynomial of order p_c:

$$\rho_i(l) = (\rho_{i,p_c})^T l^{p_c} + \cdots + (\rho_{i,1})^T l + (\rho_{i,0})^T \tag{6}$$

where l^k represent the element-by-element power of k of vector l. The controller can be completely defined using only the vectors of real parameters $\rho_{i,p_c}, \ldots, \rho_{i,1}, \rho_{i,0}$.

Following the previous parametrization, a PID controller, with a quadratic dependence from the scheduling variable can be synthesized as follows:

$$\rho^T(l) = [K_p(l), K_i(l), K_d(l)] \quad (7)$$

$$\phi^T(s) = [1, \frac{1}{s}, \frac{s}{1+Ts}] \quad (8)$$

where T is the time constant of the noise filter; as said before, considering a second order dependence from the scheduling variable l, the controller parameters $\rho(l)$ are then:

$$\begin{aligned} K_p(l) &= K_{p,0} + K_{p,1}l + K_{p,2}l^2 \ , \\ K_i(l) &= K_{i,0} + K_{i,1}l + K_{i,2}l^2 \ , \\ K_d(l) &= K_{d,0} + K_{d,1}l + K_{d,2}l^2 \end{aligned} \quad (9)$$

The parametrization of the controller defined before associated with a set of non-parametric models, allow to write every point of the Nyquist plot of the open-loop $L(j\omega, l_i) = \mathcal{K}(j\omega, l_i)\mathcal{F}(j\omega, l_i)$ as a linear function of the vector $\rho_i(l)$ ([15]):

$$\begin{aligned} \mathcal{K}(j\omega, l_i)\mathcal{F}(j\omega, l_i) &= \rho^T(l_i)\phi(j\omega)\mathcal{F}(j\omega, l_i) = \\ &\rho^T(l_i)\mathcal{R}(\omega, l_l) + j\rho^T(l_i)\mathcal{I}(\omega, l_i) = \\ &(M\bar{l}_i)^T\mathcal{R}(\omega, \bar{l}_i) + j(M\bar{l}_i)^T\mathcal{I}(\omega, \bar{l}_i) \end{aligned} \quad (10)$$

where

$$M = \begin{bmatrix} (\rho_{1,p_c})^T & \dots & (\rho_{1,1})^T & (\rho_{1,0})^T \\ \vdots & \vdots & \ddots & \vdots \\ (\rho_{n_p,p_c})^T & \dots & (\rho_{n_p,1})^T & (\rho_{n_p,0})^T \end{bmatrix}$$

$$\bar{l}_i = \left[l_i^{p_c} \dots l_i \ \vec{1} \right],$$

with $\mathcal{R}(\omega, l_l)$ and $\mathcal{I}(\omega, l_i)$ defined as the real and the imaginary part of $\phi(j\omega)\mathcal{F}(j\omega, l_i)$.

The system is now fully defined, and some optimization in terms of performance and robustness, can be performed.

3) Optimization for performance: Once the structure of the controller is defined, the optimization problem aim to find the controller parameters which are able to satisfy the following performance indexes:

- The system must remain stable for each variation, within a range, of the scheduling parameter. These constraints can be called *robustness constraints*.
- The Integrated Error (*IE*) must be reduced at its minimum. These constraints can be named *performance constraints*.

Solving the following minimization problem permits to satisfy the previous indexes:

$$\begin{aligned} \max_{M} \quad & K_{min} \\ \text{s.t.} \quad & \left(M\bar{l}_i\right)^T (\cot\alpha\mathcal{I}(\omega_k, l_i) - \mathcal{R}(\omega_k, l_i)) + K_r \leq 1 \\ & for\ all\ \omega_k, \quad i = 1, \dots, m \\ & \sum_{j=1}^{n_p} \gamma_j\rho_j(l_i) - K_{min} \geq 0 \quad for\ i = 1, \dots, m \end{aligned} \quad (11)$$

where M is the matrix of the controller parameters and K_{min} is a term used to ensure the maximization of the low frequency part of the controller, represented by the term $k_0 = \sum_{j=1}^{n_p} \gamma_j\rho_j(l_i)$. The parameters γ_j allow to express k_0 as a linear combination of $\rho(l)$ in order to keep the formulation convex. For further details, see [9], [13].

The design variables are K_r, which is linked to the gain margin value, and α, whose value is related to the phase margin (as described later in this Section). In Equation (11), the first constraints are related to the robustness performance, while the second type defines a constraint on the performance. Notice that the performance constraints focus on disturbance rejection, which is our goal. For this reason, the low-frequencies components of the controller have to be maximized ([16]).

The robustness constraints guarantee that the Nyquist plot of the open loop system will be below a line b that divide the complex plane in two regions, as visible in Figure 3. The line crosses the real axis in $-1 + K_r$ with $0 < K_r < 1$ and with an angular coefficient defined by the value of $\alpha \in]0° \ 90°]$. Ensuring that the Nyquist contour will be below the line b has the same meaning of ensuring that the open loop Nyquist plot wil not encircle the critical point $(-1, j0)$. In this manner, exploiting the Nyquist criterion ([17]), it is possible to assure asymptotic stability against slow variation of the scheduling parameter.

Furthermore, placing the Nyquist curve of the open-loop transfer function on the right side of b, ensures lower bounds on conventional robustness margins ([9]):

$$G_m \geq \frac{1}{1 - K_r} \quad (12)$$

$$\phi_m \geq \arccos\left((1 - K_r)\sin^2\alpha + \cos\alpha\sqrt{1 - (1 - K_r)^2\sin^2\alpha}\right) \quad (13)$$

$$M_m \geq K_r \sin\alpha \quad (14)$$

Where G_m, ϕ_m and M_m are the gain margin, the phase margin and the modulus margin.

As said before, α and K_r, are the design variables of the controller and their values highly influence the system performance. A wise decision of their values will be subject to discussion.

Analyzing the maximization problem presented in (11), it appears that the number of constraints depends on the frequency points ω and on the range of the scheduling parameter. Due to that, in order to solve the problem these two variables must be bounded. In particular, the problem related to the scheduling parameter is easy to solve since it is obvious that the set of non-parametric models available defines the length of the vector l_i.

The problem related to the frequency points, by the way, it is still unresolved since they are infinite. A solution to that is gridding the frequency domain: first, the band of the system must be analyzed, and then in that band, a finite number

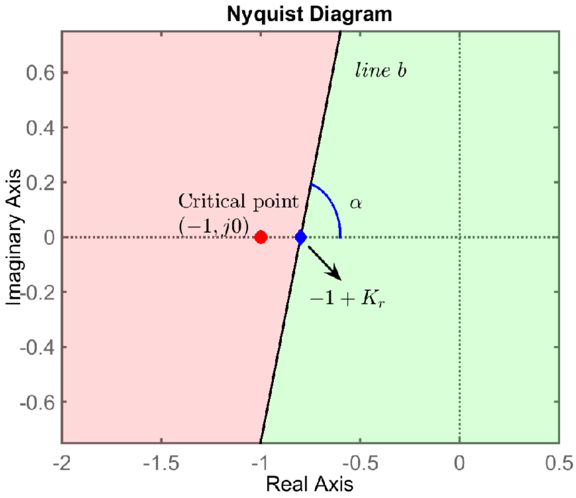

Fig. 3. Definition of the parameters used during the control design. The line b, defined by the parameter K_r and the angle α, divides the complex plane in two areas. The green area, below the line b is considered safe, while the red one, over the line b has to be avoided in order to keep the stability of the system.

of equally spaced points is taken, making the number of constraints finite. Notice that the best discretization of the frequency axis is a trade-off choice between computational load and accuracy. However, this choice is strongly depending on the shape of the frequency response of the system and a general rigorous way to grid the frequency axis is still object of ongoing research.

B. Tuning of α

The angle α is the one by which the line b crosses the real axis defining the area where the Nyquist plots need to stay. The value of this parameter has a relevant role inside the tuning of the controller, leading to an increment or decrement of the performance.
The other design variable K_r is directly connected to the Gain Margin of the closed loop system by Equation (12). Once this performance index is fixed, the others (module margin and phase margin) can be decided and consequentially even the value of α can be chosen. Instead of maximizing the phase margin, our approach is different: it is important to remove the sway as fast as possible, even permitting overshoot in the angle. For this reason it has been decided to fix the gain margin in order to obtain a robust controller and then compute the value of α by minimizing the time response to the sway disturbance. Summarizing, the following procedure is adopted.

1) Grid the parameter α within its range;
2) Tune a controller for each value of α by solving the constrained optimization problem (11);
3) Evaluate the settling time t_s of the closed-loop system for each α, where t_s is defined as the time elapsed from the application of an ideal instantaneous step input to the instant at which the output has entered and remained within a symmetric error band of 5%;
4) Choose the α which minimizes the mean of the settling time t_s (over the scheduling parameter l)

$$J_\alpha = \frac{1}{n_l}\sum_{i=1}^{n_l} t_s(\alpha, l_i).$$

An alternative to the minimization of the mean of the obtained settling times is the minimization of the worst case. In that sense, the following cost could be used in place of J_α:

$$V_\alpha = \max_i t_s(\alpha^{(i)}).$$

IV. Experimental Results

In this section the results achieved on the real bridge crane will be presented. First, two different controller tuning processes will be presented and tested in simulation. Then, the same controllers will be tested on the real system.

A. Controller tuning

Two different controllers are tuned: a time-invariant one based on the model at $l = 4.5$ m and a gain-scheduling one following the method described in Sec. III.
The controller structure has been defined as

$$\mathcal{K}(s,l) = P_1(l)\frac{1}{1+Ts} + P_2(l)\frac{s}{1+Ts} \tag{15}$$

where P_1 and P_2 have a quadratic dependence on the scheduling parameter l: $P_i(l) = P_{i,2}l^2 + P_{i,1}l + P_{i,0} \quad i = 1, 2$. The selected structure arises from various consideration about the aim of the controller and the model of the system. First, the controller was chosen without a pure integral part since the cancellation with the derivator in the transfer function may hide some unstable behavior. For this reason, a pole in low frequency has been added; this pole provides also high gain at low frequency, increasing the disturbance rejection. A zero is used to increase the phase of the system. Finally, the controller has a relative degree equal to zero, avoiding the introduction of delay in the loop.

The only parameter that must be chosen for the tuning of the controller is K_r. A good trade-off between robustness and performance is found to be $K_r = 0.2$, which is then set as our design parameter.

1) Time Invariant controller - K_{TI}: To tune a time-invariant controller, the structure in (15) is modified as

$$\mathcal{K}(s) = \bar{P}_1\frac{1}{1+Ts} + \bar{P}_2\frac{s}{1+Ts}. \tag{16}$$

Using the method described in Section III, the controller does not guarantee robustness and optimal performance for all the variation of l, but only for $l = 4.5$m. This leads to a loss of performance for the other values of the scheduling parameter.

The bridge crane used for the tests has a rope length which spans from 1 to 6.5 m, leading to a frequency bandwidth going from 0.19 Hz to 0.49 Hz. For this reason, the frequency limits has been set from 0.01 Hz to 10 Hz, gridded every 0.001 Hz, leading to 9991 frequency points.

The gain margin K_r, has been set equal to 0.2, while the other design variable, α, as described in III-B, has been

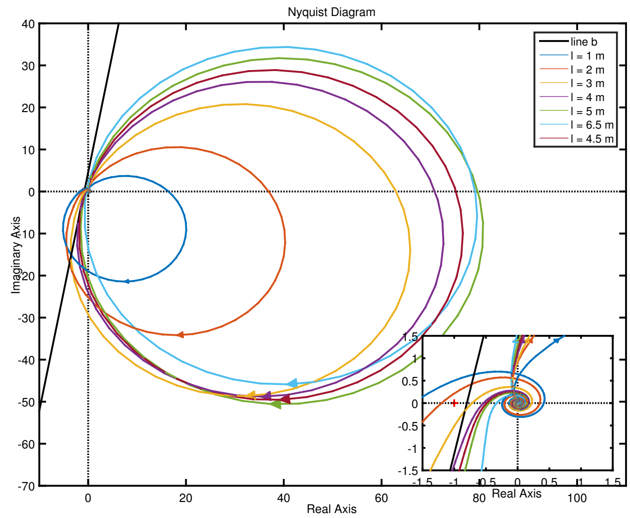

Fig. 4. Nyquist plots of the system with the K_{TI} controller. The stability is guaranteed only for $l = 4.5m$.

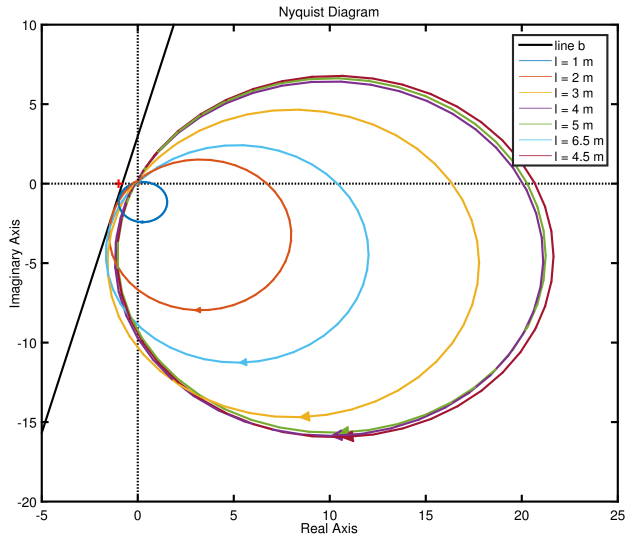

Fig. 5. Nyquist plots of the system with the Gain Scheduling controller. All the lines remain bounded by the line b.

tuned evaluating the step response time; in particular the best performance are achieved for $\alpha = 80°$. These values lead to a bound in the gain margin of 1.25 and a phase margin of 28°. We stress that these values are valid only for the controller tuned at 4.5 m.

The obtained controller parameters are: $\bar{P}_1 = -2.704$, $\bar{P}_2 = -9.882$.

Figure 4 shows the Nyquist diagrams of the loop transfer function with the previously computed controller. In particular the Nyquist contour is presented for different rope lengths: from 1 to 6.5 m. Since during the tuning phase only the model at 4.5 m has been considered, the constraints related to performance and robustness may not be respected. In particular it is clear how the Nyquist contour exceeds the line b in 4 of the 6 different models (without considering the one at 4.5m). More in deep, the controller for the model at 1 and 2 m pushes the Nyquist diagram to rotate around the critical point $(-1, j0)$ making the closed loop system unstable.

2) Gain Scheduling - K_{GS}: The gain scheduling has been tuned exploiting six different identified model at different rope length. In particular the rope length varied from 1 to 6.5 with 6 almost equispaced steps. The controller has the same structure of the one described in Sections IV-A.1 and III. The two parameters of the controller have a quadratic dependence on the scheduling parameter.

The frequency band is the same of the K_{TI} controller, so the number of constraints related to the robustness index for each rope length is still the same, 9991. The difference here is that, instead of only one scheduled parameter, there are 6 different values of the parameter. This leads to $9991 \cdot 6 = 59946$ constraints. The other type of constraints, the performance ones, are related only to the number of values assumed by the scheduling parameter, so only 6, leading to a total number of 59952 constraints.

The optimization problem leads to these controller parameters: $P_1(l) = -0.004 \cdot l^2 + 0.016 \cdot l - 2.66$, $P_2(l) = 0.433 \cdot l^2 - 3.338 \cdot l + 1.451$. These parameters were obtained with an $\alpha = 75°$, which permits to minimize the step response time, and a $K_r = 0.2$. These values permits to have a gain margin equal to 1.25 and a phase margin of 24.4°. These indexes represent a lower bound for all the different values assumed by the scheduling parameter l. With such a controller, as visible in Figure 5, all the Nyquist plots for different rope lengths are in the safe area, below the line b defined by the parameters α and K_r.

In Figure 6, the step response of the K_{TI} *controller*, the K_{GS} *controller* and the system without control are shown. It can be observed that the K_{TI} controller, at its tuning point 4.5 m, has better performance compared to the Gain Scheduling controller.

At 6 m the time invariant controller still have better performance compared to K_{GS}, but the performance loss in the Gain Scheduling case is due to the high level of robustness requested, which is not granted by K_{TI}. In the 3 m case instead, the damping of the sway is more similar.

B. Experimental results

In this subsection, some tests on the bridge crane described in Section II are presented, to evaluate the performance of the two controllers from a comparative perspective. Three tests at different rope length are made:

- Test 1: step response of the system with $l = 3\ m$;
- Test 2: step response of the system with $l = 4.5\ m$;
- Test 3: step response of the system with $l = 6\ m$;

The input of the system, in these three tests, is not a real step, since it is physically impossible to implement a real step on a mechanical system like the bridge crane. Due to the high inertia and to some structural limitations it was possible to

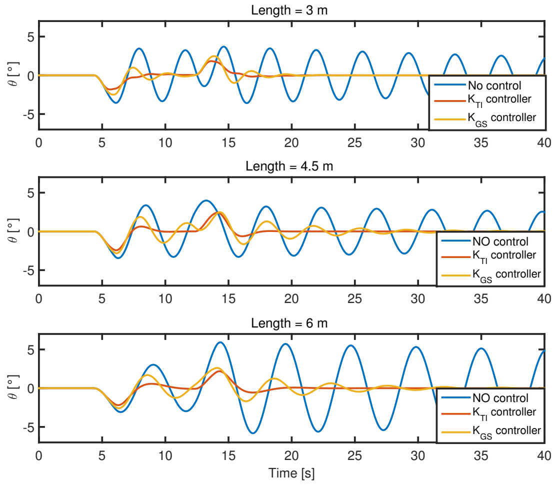

Fig. 6. Simulation results of a step response of the closed loop system without control, with the time invariant and with the gain scheduling controller, for three different values of the rope length.

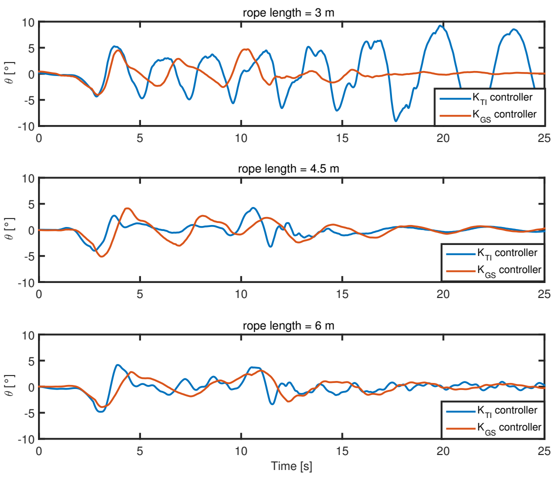

Fig. 7. Real tests made on the bridge crane. The results are shown for the system with the time invariant controller and with the gain scheduling controller, for three different values of the rope length.

use as input only a ramp that reach the maximum speed in 1 second. Higher acceletarions introduce slipping of the wheel on the track and are outside the nominal operation range of the system.

The results of these tests can be seen in Figure 7. These tests show the effectiveness of the Gain Scheduling controller, which is able to attain the aim of reducing the sway of the load in all the conditions. K_{GS} is more robust compared to the K_{TI} one; in fact, in the *Test 1* the system with the K_{TI} controller is unstable.

These results are confirmed by computing the *RMS* of the oscillations. K_{TI} has *RMS* values of 4.89, 1.23 and 1.40 respectively at 3, 4.5 and 6 m. K_{GS} shows almost constant values: 1.49, 1.32 and 1.35 at the same rope length values.

V. Conclusions

In this paper, the problem of sway reduction in bridge cranes is tackled. To this aim, a fixed-order gain scheduling controller is designed, with the aim of being robust with respect to unmodeled dynamics and maximizing the speed performance. The original tuning method has been extended by adding a performance oriented tuning of α and the resulting controller has been experimentally validated and compared with a time-invariant law tuned according to the same specifications. The proposed control algorithm, thanks to its low computational burden, can be implemented on a low cost hardware.

Future work will be devoted to LPV control of bridge cranes for fast load lifting.

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