

Modeling and Identification of an Electro-Hydrostatic Actuator

D. Belloli*, F. Previdi#, S.M. Savaresi§, A. Cologni*, M. Zappella#

* *Consorzio Intellimech, Parco Scientifico e Tecnologico Kilometro Rosso
Via Stezzano, 87 24100 Bergamo, Italy*

*Dipartimento di Ingegneria dell'Informazione e Metodi Matematici,
Università degli Studi di Bergamo, Via Marconi, 5 24044 Dalmine (BG), Italy*

§ *Dipartimento di Elettronica e Informazione, Politecnico di Milano
P.zza Leonardo da Vinci, 32 20133 Milano, Italy*

Abstract: The aim of this paper is to develop an accurate model of an electro-hydrostatic actuator (EHSA) and to estimate the model parameters. The physical behavior of each components has been analyzed and a linear model has been obtained. Then, the cylinder friction phenomenon and the cylinder chambers volume variations have been considered and a non linear model has been developed. An EHSA prototype has been produced and used to estimate the parameters and to validate the models. The comparisons between simulation results and experimental data show that the linear model well describes the main dynamics of the EHSA and the non linear model fits the data with an high accuracy.

Keywords: Hydrostatic Actuator, Model, Identification, Friction

1. INTRODUCTION

Electro Hydraulic Actuators (EHA) are utilized in many applications of several industry sectors, i.e. the aircraft control surfaces movement in aeronautics, the disk brake system in automotive and the materials press in manufacturing machines [1-2]. The wide range usage of this technology is mainly due to the high force/torque developed with remarkable reliability. However, the hydraulic systems present a low energy efficiency, due to the continuous running of the pump in order to keep a constant supply pressure, and need a complex control structure because of the non linearity of the electro-valve [3-4].

The technological alternative, largely used in the automation industry, is represented by the Electro Mechanical Actuators (EMA) [5]. These systems are characterized by an elevated positioning precision, owing to an efficacious control strategy [6], and high speed dynamics, due to the wide velocity range. Nevertheless, the use of the EMA is restricted to low-medium power applications because of the reduced force range and the reliability of the mechanical components, that could be crucial in safety-critical applications.

In latest years, a new hydraulic actuation concept has been conceived and developed, producing a new generation of actuators: the Electro HydroStatic Actuators (EHSA) [7-8]. This technology is based on a closed-circuit hydraulic transmission, composed by a bidirectional pump, driven by an electrical motor, that regulates the oil movement and the pressure difference in the chambers of an hydraulic cylinder (Fig.1). The EHSA technology provides the main advantages of both EHA and EMA, reducing most of the drawbacks. Actually, the hydraulic circuit allows to reach an high force range and a very good reliability, as the EHA, while the

presence of the coupled electric motor and pump makes possible an accurate control and positioning and large bandwidth dynamics, as the EMA. Moreover, the EHSA presents also an high energy efficiency, due to the fact that the pump works only on a movement demand and the actuating power is transferred by electricity (Power by Wire) instead of by the oil in the pipes (Power by Pipe). For these reasons, the EHSA has been lately applied in aeronautic sector, particularly in the critical safety applications [9].

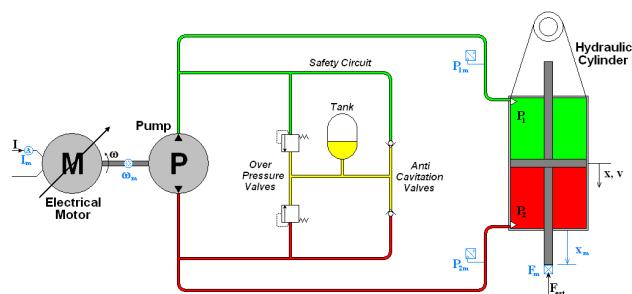


Fig. 1. Schematic representation of an EHSA actuator

In order to design accurate control strategies and to understand the dynamic behavior of the system, a detailed model of the EHSA is useful. Different models are proposed in [10-13]. Some of them describe the behavior of each components in order to analyze the dynamics of the complete EHSA system and the remaining, which target is to design a control strategy, consider the global transfer function from the command inputs (electrical motor current or speed) and the measurable outputs (actuator speed or position).

In this paper, an EHSA prototype, designed and assembled by the Laboratorio di Meccatronica of the Politecnico di Torino, will be described. This is used as the test bench to which this work is referred. First of all, linear modelization of

the physical behavior of each single component of the actuator has been performed and a linear model of the actuator has been developed. Then, nonlinear phenomena, such as friction and cylinder chambers volume variations, have been considered and introduced in the model. Finally, model parameters have been tuned using results from experiments carefully designed. The models with the estimated parameters have been validated performing experiments in normal operating conditions.

The paper is organized as follows. In Section II, the EHSA layout and the details of the experimental equipment are illustrated. In Section III, a linear mathematical model of each single component is presented together with the overall linear model of the actuator. In Section IV, the main nonlinear phenomena have been introduced in the model. Finally, the parameter estimation and experimental validation is reported and discussed in Section V.

2. EXPERIMENTAL SET UP

The layout of the EHSA considered in this paper is based on a closed-circuit hydraulic transmission, which main components are: a DC brushless electrical motor directly connected to a bidirectional fixed displacement gear pump; an hydraulic cylinder with a through rod connected to the pump; and a safety circuit, composed by a small tank, anti-cavitation and over pressure valves. The specifications of the single components of the EHSA are reported in Tab. 1.

Table 1. Specification of the EHSA components

| Component | Features | Unit | Value |
|--------------------|----------------|------------------------|-------|
| Electrical Motor | Power | [kW] | 1.58 |
| | Speed (max) | [rpm] | 5000 |
| | Torque (max) | [Nm] | 16.1 |
| Bidirectional Pump | Displacement | [cm ³ /rev] | 3.7 |
| | Speed (max) | [rpm] | 4500 |
| | Pressure (max) | [bar] | 210 |
| Hydraulic Cylinder | Stroke | [mm] | 200 |
| | Bore | [mm] | 63 |
| | Rod Diameter | [mm] | 45 |

Rotation of the motor-pump assembly determines oil flow in the cylinder chambers and an increase of the differential pressure in the hydraulic cylinder. The pressure applied to the piston surface generates a force acting on the actuator rod.

The safety circuit is necessary because the pressure in the hydraulic circuit, in particular operating conditions, could excessively increase or decrease, at risk of pump cavitation and damage. The safety circuit is made by over-pressure valves and anti-cavitation valves that give access to a small oil tank to restore the hydraulic circuit pressure to the nominal range. The assembled system is shown in Fig. 2.

The EHSA prototype has been mounted on a dynamic test bench consisting in a mechanical structure to accommodate the actuator and passive load represented by an elastic spring (Fig. 3). The experimental bench is completed by the electric cabinet, containing the power drive unit to command the electrical motor and electronic unit for the sensors acquisition and data-logging.

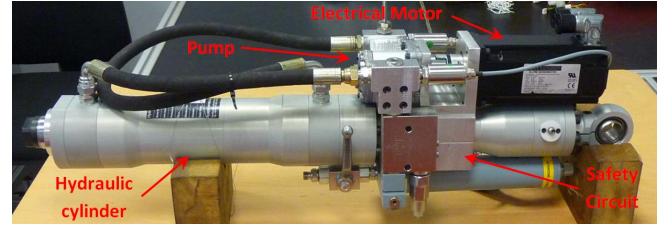


Fig. 2. The EHSA prototype, (LxWxH = 80x15x25 cm)

The EHSA is endowed by amperemeter and encoder to measure the current and the speed of the electrical motor; two pressure sensors to monitor the pressures at the inlet and outlet ports of the pump; linear potentiometer and load cell to acquire the actuator position and force. All the sensor signals, sampled at 250Hz, are acquired by the electronic control board Actua EKU (based on Texas Instruments DSP TMS320F2818), which also generates and provides the reference signal utilized by the power drive to command the electrical motor.



Fig. 3. The EHSA mounted on the test bench, (LxWxH = 300x55x75 cm)

3. LINEAR MODEL OF THE SYSTEM

In this section the linear model of the EHSA is presented. The main features and the physical behavior of the electrical motor, bidirectional pump and hydraulic cylinder have been taken into account. Then, considering the assemblage of the single components, the overall linear model has been deduced.

3.1 Electrical Motor

The motor dynamics has been described by a standard model for DC motors based on the mechanical torque balance, represented in the following equation

$$J_M \cdot \dot{\omega}_M + B_{FM} \cdot \omega_M + T_L = K_T \cdot I_M \quad (1)$$

where I_M is the supplied current, which is regulated by a closed loop control system managed by the driver; K_T is the torque constant of the motor; ω_M is the motor shaft rotation speed; J_M represents the total inertia of motor and load; B_{FM} is a viscous friction coefficient and T_L is the load torque.

3.2 Bidirectional Pump

To realize the EHSA prototype, a fixed displacement gear pump in bidirectional rotation configuration has been directly connected to the electrical motor. The pump flow Q_P and pump torque T_P are defined by (2) and (3),

$$Q_P = D \cdot \omega_P - K_{LP} \cdot (P_{IN} - P_{OUT}) \quad (2)$$

$$T_P = D \cdot (P_{IN} - P_{OUT}) \quad (3)$$

where ω_p is the pump rotation speed; D is the pump displacement; K_{LP} is the pump leakage coefficient; P_{IN} and P_{OUT} are respectively the inlet and outlet port pressure.

3.3 Hydraulic Cylinder

The hydraulic cylinder, realized in high resistance steel, is separated in two chambers by a double effect piston, which is connected to the chromate steel rod passing through one of the cylinder head to permit the load connection. The oil flow continuity equations of the chambers and the force balance of the hydraulic cylinder are showed in (4.a), (4.b) and (5):

$$Q_A = A \cdot v + \frac{V_A}{\beta} \cdot \dot{P}_A + K_{LC} \cdot (P_A - P_B) \quad (4.a)$$

$$Q_B = A \cdot v - \frac{V_B}{\beta} \cdot \dot{P}_B + K_{LC} \cdot (P_A - P_B) \quad (4.b)$$

$$A \cdot (P_A - P_B) = M \cdot \dot{v} + B_{FR} \cdot v + F_{EXT} \quad (5)$$

where P_A , Q_A and V_A and P_B , Q_B and V_B are respectively the oil flows, the pressures and the volumes of the two cylinder chambers; A is the piston surface, which is supposed identical for both chambers; v is the rod velocity; β is the bulk modulus of the hydraulic oil; K_{LC} is leakage coefficient of the hydraulic cylinder; M is the load mass applied to the actuator; B_{FR} is the viscous friction coefficient of the rod and F_{EXT} is the external force applied to the actuator.

3.4 EHS complete model

Since the electrical motor shaft is rigidly coupled with to the pump shaft, the rotation speeds ω_M and ω_p can be considered equal and they are renamed ω , the system rotation speed. Similarly, under the rigid connection hypothesis, the electrical motor load torque T_L corresponds to the pump torque T_p . So, into (1) and (2), it will be set:

$$\omega = \omega_M = \omega_p$$

$$T_L = T_p$$

The hydraulic cylinder chambers are directly connected to the inlet and the outlet ports of the pump with short pipes with negligible pressure drop over they length. So, the flows Q_p , Q_A and Q_B can be considered equal and they are renamed as Q . For the same reason, the pressure P_{IN} at the inlet port of the pump is equal to the pressure P_A inside the "A" chamber of the cylinder and, similarly, P_{OUT} at the output port of the pump and P_B inside the "B" chamber of the cylinder can be regarded as equal. So, into (2-5), it will be set:

$$Q = Q_p = Q_A = Q_B$$

$$P_1 = P_A = P_{IN}$$

$$P_2 = P_B = P_{OUT}$$

Then, the leakage of the pump K_{LP} and of the hydraulic cylinder K_{LC} have be combined in a general leakage term K_L :

$$K_L = K_{LP} + K_{LC}$$

Finally, under the aforementioned hypotheses, by substitution of (3) into (1) and (2) into (4), (1-5) can be re-written as follows:

$$\begin{cases} \dot{\omega} = \frac{1}{J_M} \cdot (K_T \cdot I_M - B_{FM} \cdot \omega - D \cdot (P_1 - P_2)) \\ \dot{P}_1 = \frac{\beta}{V_1} \cdot (D \cdot \omega - A \cdot v - K_L \cdot (P_1 - P_2)) \\ \dot{P}_2 = \frac{\beta}{V_2} \cdot (-D \cdot \omega + A \cdot v + K_L \cdot (P_1 - P_2)) \\ \dot{v} = \frac{1}{M} \cdot (A \cdot (P_1 - P_2) - B_{FR} \cdot v - F_{EST}) \end{cases} \quad (6)$$

The model (6) can be further simplified assuming that the hydraulic cylinder chambers volumes are equal. Under this hypothesis, it is possible to introduce single pressure variable ΔP , which is differential pressure in the hydraulic cylinder.

$$\Delta P = P_1 - P_2$$

So, system (6) can be rewritten as:

$$\begin{cases} \dot{\omega} = \frac{1}{J_M} \cdot (K_T \cdot I_M - B_{FM} \cdot \omega - D \cdot \Delta P) \\ \dot{\Delta P} = 2 \frac{\beta}{V} \cdot (D \cdot \omega - A \cdot v - K_L \cdot \Delta P) \\ \dot{v} = \frac{1}{M} \cdot (A \cdot \Delta P - B_{FR} \cdot v - F_{EXT}) \end{cases} \quad (7)$$

The system described in (7) is a third order dynamical system, with

- two inputs: the current I_M , which will act as a control variable; the external load force F_{EST} , which is an external disturbance;
- three state variables, namely the system rotation speed ω , the differential pressure ΔP and the speed v of the cylinder rod;
- two measured outputs: the system rotation speed ω and the rod speed v , obtained measuring the rod position.

The linear model could be useful in order to describe the main dynamic behavior of the system and to design the control strategy. But preliminary tests show the presence of non linear phenomena, in particular regarding the friction in the electrical motor and hydraulic actuator. An increasing current profile has been applied to the electrical motor (Fig.4). It is worth to notice that motor begin to rotate (green line) only when the current exceeds a specific threshold, corresponding to the torque to overcome the initial friction. Similarly, the actuator rod needs an overshoot of pressure, corresponding to force overshoot, to start to move (red line). These behaviors indicate the presence of non linear friction, so a more detailed model is necessary to correctly describe these phenomena.

4. NON LINEAR MODEL OF THE SYSTEM

In the linear model, only viscous friction is considered both in the electrical motor and in the hydraulic cylinder. In the electrical motor this approximation is fairly realistic because the motor shaft is mounted on ball bearing in order to minimize the friction. On the contrary, in the hydraulic cylinder, taking into account only the linear friction is considerably restrictive. In fact, the piston is equipped with a rubber seal to separate the two cylinder chambers and to avoid excessive internal leakage. A similar seal is installed on the cylinder head close to the rod hole. The more the seals are

efficient, the more is the friction amount that the overall EHSA opposes to the movement [14]. The presence of non linear friction in the EHSA system is also confirmed by the experimental results showed in Fig. 4.

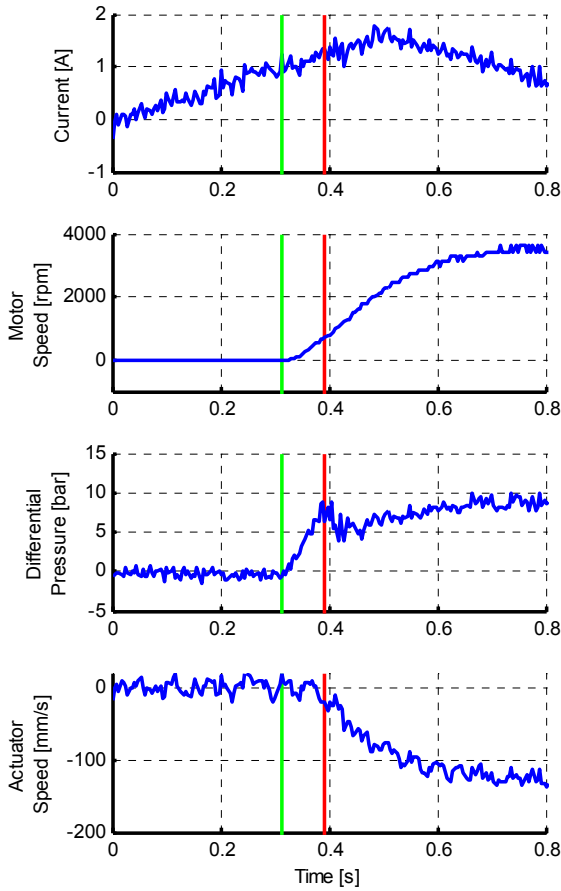


Fig. 4. Presence of non linear friction.

For these reasons, a more detailed model of the friction has been considered. Several static and dynamic non linear models of the friction phenomenon are proposed in [15-16]. In the present work, the static friction and the Coulomb friction [20] have been added, as generally illustrated in Fig.5.

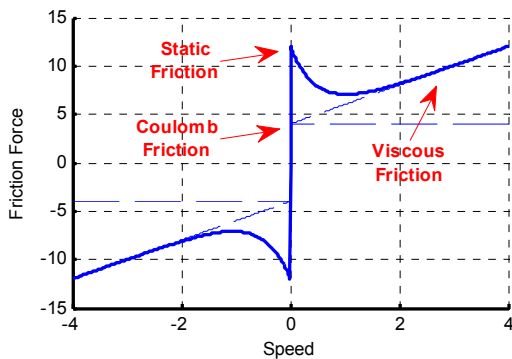


Fig. 5. Representation of the friction function.

The total friction functions are described in (8). Adding the non-linear friction terms, the torque balance in the electrical motor (1) and the force balance in the hydraulic cylinder (5) result modified respectively as in (9) and (10).

$$T_{FM}(\omega) = A_{FM} \cdot \text{sign}(\omega) + B_{FM} \cdot \omega + C_{FM} \cdot \text{sign}(\omega) \cdot e^{-\frac{|\omega|}{\tau_{FM}}} \quad (8)$$

$$F_{FR}(v) = A_{FR} \cdot \text{sign}(v) + B_{FR} \cdot v + C_{FR} \cdot \text{sign}(v) \cdot e^{-\frac{|v|}{\tau_{FR}}}$$

$$T_M = J_M \cdot \dot{\omega}_M + T_{FM}(\omega_M) + T_L \quad (9)$$

$$A \cdot (P_A - P_B) = M \cdot \dot{v} + F_{FR}(v) + F_{EXT} \quad (10)$$

Furthermore, in the linear model, the cylinder chambers volume values, which determine the amount of oil subject to compression in the oil flow continuity equations, have been assumed constant. The precision of the model could be improved considering the chambers volume variation as a function of the actuator position (rod stroke). Then, the oil flow continuity equations of cylinder chambers (4) have been modified as:

$$Q_A = A \cdot v + \frac{V_{A0} + A \cdot x}{\beta} \cdot \dot{P}_A + K_{LC} \cdot (P_A - P_B) \quad (11.a)$$

$$Q_B = A \cdot v - \frac{V_{B0} - A \cdot x}{\beta} \cdot \dot{P}_B + K_{LC} \cdot (P_A - P_B) \quad (11.b)$$

where x is the rod position; V_{A0} and V_{B0} are the cylinder chambers volumes at initial condition $x = 0$.

Considering the motor and cylinder frictions and the cylinder chambers volume variations, the overall non-linear EHSA (6) model becomes as in (12). It is worth noting that in this model it is not possible to merge the two equations describing the dynamics of the two hydraulic cylinder chambers pressures. Moreover, to describe the chambers volume variations, the actuator position is needed and a new dynamic variable has been added. The result is a fifth order dynamical system.

$$\begin{cases} \dot{\omega} = \frac{1}{J_M} \cdot (K_T \cdot I_M - T_{FM}(\omega) - D \cdot (P_1 - P_2)) \\ \dot{P}_1 = \frac{\beta}{V_{10} + A \cdot x} \cdot (D \cdot \omega - A \cdot v - K_L \cdot (P_1 - P_2)) \\ \dot{P}_2 = \frac{\beta}{V_{20} - A \cdot x} \cdot (-D \cdot \omega + A \cdot v + K_L \cdot (P_1 - P_2)) \\ \dot{v} = \frac{1}{M} \cdot (A \cdot (P_1 - P_2) - F_{FR}(v) - F_{EST}) \\ \dot{x} = v \end{cases} \quad (12)$$

5. PARAMETER ESTIMATION AND MODEL VALIDATION

The EHSA model presents a large number of parameters. Some of them can be deduced from the components features and datasheets, such as the pump displacement and the piston area. Anyway, most of the parameters, such as friction or leakage coefficients, have to be experimentally identified. In order to collect experimental data, several tests have been performed using the dynamic bench. The tests have been done with the unloaded actuator (the spring has been removed from the bench) in order to characterize the behavior of the EHSA without any external influence. The current driving the electric motor has been used as input of the EHSA system. The current is controlled by a closed-loop implemented by the driver and, in the estimation process, a band-limited white noise (BLWN) has been applied as current reference. The motor velocity, the hydraulic cylinder

chambers pressures and the actuator position have been measured, whereas the actuator velocity has been obtained from the actuator position. The collected data are showed in Fig. 6.

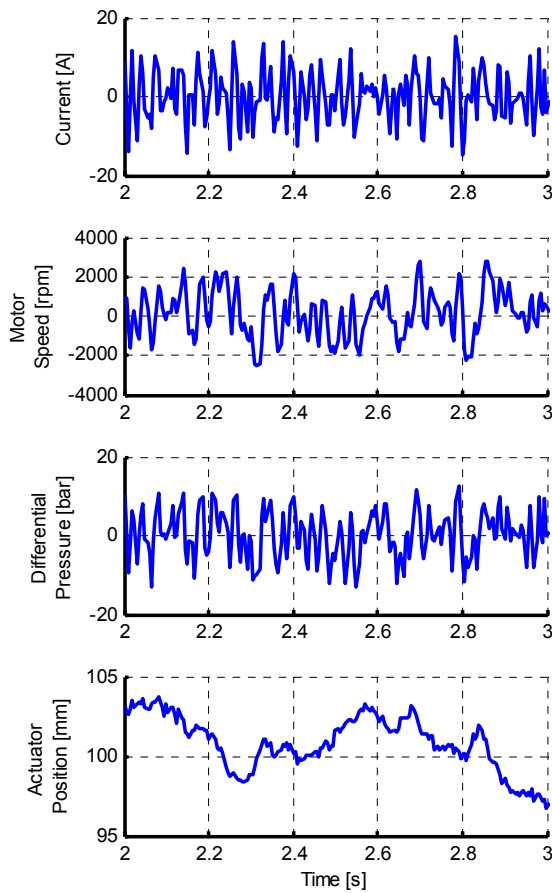


Fig. 6. I/O data used for parameter estimation

The estimation process has been processed by the minimization of the cost function $J(\theta)$ of (13), which represents the mean square error between the measured data y_{meas} and model simulation y_{model} , depending by the parameters vector θ .

$$J(\theta) = \sum_{t=1}^N [y_{meas}(t) - y_{model}(t, \theta)]^2 \quad (13)$$

Table 2. Identified Model Parameter Values

| Variable | Unit | Linear Model Value | Non Linear Model Value |
|----------|-------------------------|--------------------|------------------------|
| J_M | kg m ² | 3,652 E-04 | 3,312 E-04 |
| A_{FM} | N m | | 8 E-01 |
| B_{FM} | kg m ² / s | 1,8 E-03 | 1,35 E-03 |
| C_{FM} | N m | | 8 E-02 |
| V | m ³ | 2,042 E-3 | |
| V_{10} | m ³ | | 2,056 E-3 |
| V_{20} | m ³ | | 2,033 E-3 |
| K_L | m ³ / (Pa s) | 4,13 E-13 | 4,05 E-13 |
| M | kg | 16,46 | 16,52 |
| A_{FR} | N | | 854 |
| B_{FR} | N s / m | 9712 | 9186 |
| C_{FR} | N | | 213 |

The estimated value of the parameter for the models are shown in Tab. 2.

Both the linear and the non linear models with identified parameters have been validated using different experimental data. In particular, a sequence of positive and negative steps of amplitude 1.5A and a sinusoid of amplitude 1.5A and frequency 2Hz have been applied to the input current. The comparison between the experimental data (blue line), the linear model simulation (black line) and the non linear model simulation (red line) is showed in Fig.7 and Fig.8.

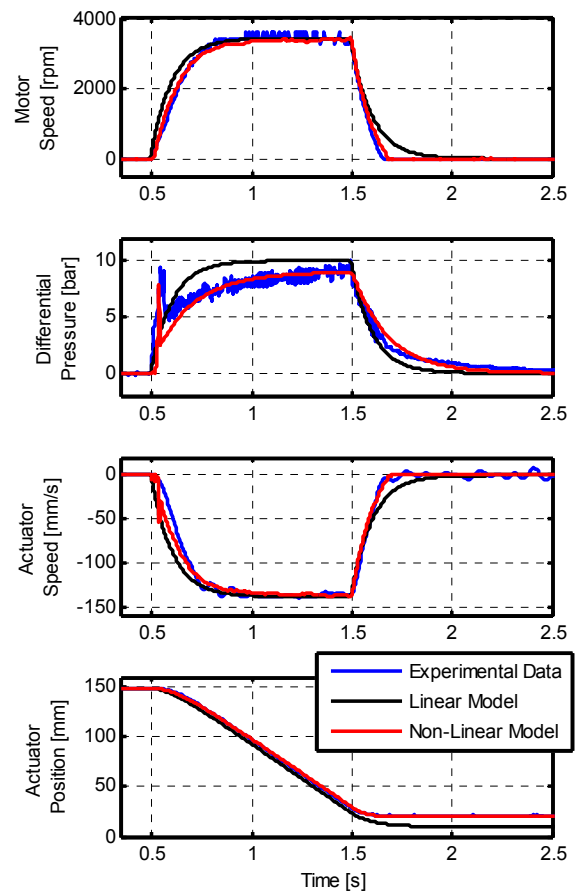


Fig. 7. Results of a validation experiment performed using a current step input

The linear model correctly captures the main dynamics of the most important variables of the EHSA, but it is not able to describe some other behaviors of the plant, such as the quick decelerations of the electrical motor and the actuator speeds as a consequence of the negative current step. Notice that the velocity estimate error determines, as expected, a steady-state error in the actuator position.

Moreover, the experimental data of the differential pressure present an initial overshoot, necessary to generate the force to overcome the static friction of the actuator rod and the linear model is not capable to describe it. On the contrary, the non linear model properly fits the experimental data, both in the pressure overshoot and in the quick decelerations of the electrical motor and actuator speeds. The correct tracking of the actuator speed is also confirmed by the absence of steady state error in the actuator position.

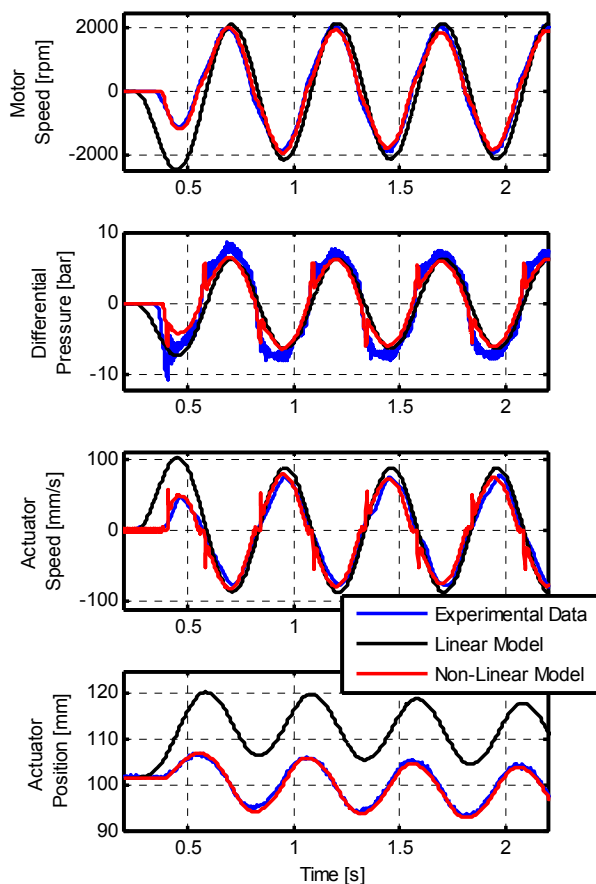


Fig. 8. Results of a validation experiment performed using a current sinusoidal input

The sinusoidal current input test underlines the notable importance of the friction: in fact, the initial half wave is lower than the others, due to the presence of static friction. The linear model is not able to model these phenomena and the consequence is an imprecise tracking of the actuator speed, which determines a steady state error in the position. The introduction of the static and Coulomb friction in the non linear model permits the an accurate fitting of the experimental data, also in the initial overshoot of differential pressure.

6. CONCLUSIONS

This paper refers to the modeling of EHSA and the estimation of the model parameters. Considering the physical behavior of the single components, a liner model able to describe the main dynamic of the EHSA has been obtained. Moreover, the cylinder friction phenomenon and the cylinder chambers volume variations have been analyzed and a non linear model has been developed. The non-linear model is able to describe the EHSA behavior with a notable accuracy both in steady-state and dynamic conditions. The efficacy of the both developed models is proved by the comparison between simulation results and experimental data.

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