

# A novel Control strategy for Semi-Active suspensions with variable damping and stiffness

Cristiano Spelta, Fabio Previdi, Sergio M. Savaresi, Paolo Bolzern,  
 Maurizio Cutini, Carlo Bisaglia.

**Abstract**—The problem considered in this paper is the study and the control strategy design of semi-active suspensions featuring the regulation of both damping and stiffness. This work presents an evaluation of the performances and drawbacks achieved by such suspension architecture, also in a non-linear setting (explicitly taking into account the stroke limits of the suspension). This paper then proposes a new comfort-oriented variable-damping-and-stiffness control algorithm, named Stroke-Speed-Threshold-Stiffness-Control (SSTSC), which overcomes the critical trade-off between the choice of the stiffness coefficient and the end-stop hitting. The use of a variable-damping-and-stiffness suspension, together with this algorithm, provides a significant improvement of the comfort performances, if compared with traditional passive suspensions and with more classical variable-damping semi-active suspensions.

## I. INTRODUCTION

THE topic of this paper is control strategy design for a controllable suspension with variable damping and stiffness. While the modulation of the damping coefficient is commonly used and can be easily obtained with different technologies (see e.g. [4], [2], [5], [9], [15], [21], [20], [18], [17], [13], and references cited therein), the control of the spring stiffness is a much more subtle and elusive problem. Load-leveling or active suspensions based on hydro-pneumatic or pneumatic technologies are subject to spring-stiffness variations, but this is more a side-effect than a real control variable ([3], [5], [12]).

This work contains (to the best of our knowledge) the following innovative contribution: a detailed analysis on the advantages and trade-offs of a variable-damping-and-stiffness suspension systems is developed. An innovative control strategy suited to variable-damping-and-stiffness

suspensions is proposed. The algorithm presented herein is named Stroke-Speed-Threshold-Stiffness-Control (SSTSC): it based on the recently developed Mixed SH-ADD rationale [17].

In this work the control objective is the minimization of the vertical acceleration of the vehicle ([9], [21], and reference cited therein).

## II. SEMI-ACTIVE SUSPENSION MODELS WITH VARIABLE STIFFNESS AND DAMPING.

This section is devoted to the introduction and comparison of two quarter car models as reported in Fig.1 (see for details e.g. [9], [21]). The first (IS model) describes an ideal suspension system with variable damping and stiffness. The other architecture (DSS model), based on semi-active devices, can approximate the ideal system, and can be implemented in practice: the architecture based on passive devices was previously introduced in [11] and was generalized to a semi-active framework in a recent work([19]).

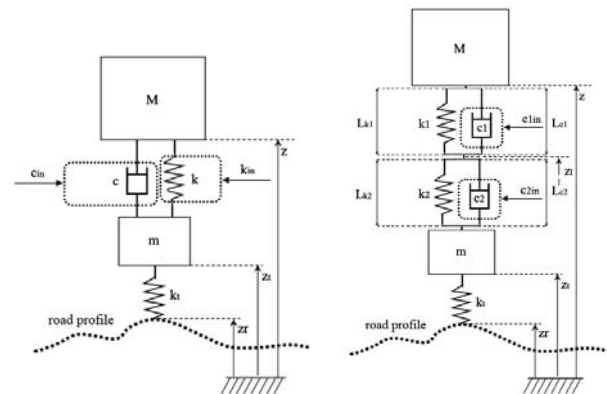


Fig.1 Quarter-car suspensions systems. From left to right: ideal suspension with variable damping and stiffness (IS); double suspension system (DSS).

*Quarter-car model of an Ideal Suspension (IS) with variable stiffness and damping:*

$$\begin{cases} M\ddot{z}(t) = -c(t)(\dot{z}(t) - \dot{z}_t(t)) - k(t)(z(t) - z_t(t) - \Delta_s) - Mg \\ m\ddot{z}_t = +c(t)(\dot{z}(t) - \dot{z}_t(t)) + k(t)(z(t) - z_t(t) - \Delta_s) + \\ \quad -k_t(z_t(t) - z_r(t) - \Delta_t) - mg \\ \dot{c}(t) = -\beta c(t) + \beta c_{in}(t) \quad c_{min} \leq c_{in}(t) \leq c_{MAX} \\ \dot{k}(t) = -\gamma k(t) + \gamma k_{in}(t) \quad k_{min} \leq k_{in}(t) \leq k_{MAX} \end{cases} \quad (1)$$

This work has been partially supported by MIUR project “New methods for Identification and Adaptive Control for Industrial Systems”.

C. Spelta and F. Previdi are with the Dipartimento di Ingegneria dell’Informazione e Metodi Matematici, Università degli Studi di Bergamo, viale Marconi 5, 24044 Dalmine (BG) ITALY (e-mail: cristiano.spelta@unibg.it; fabio.previdi@unibg.it).

M. Cutini and C. Bisaglia are with CRA-ING, Laboratorio di Treviglio, via Milano 43, 24043 Treviglio (BG) ITALY (e-mail: maurizio.cutini@entecra.it; carlo.bisaglia@entecra.it).

S. M. Savaresi and P. Bolzern are with the Dipartimento di Elettronica e Informazione, Politecnico di Milano, P.zza L. da Vinci 32, 20133 Milano ITALY (e-mail: savaresi@elet.polimi.it; bolzern@elet.polimi.it; delvecchio@elet.polimi.it).

Quarter-car model of a Double Suspension System (DSS):

$$\begin{cases} M\ddot{z}(t) = -k_1(z(t) - z_l(t) - \Delta_{s1}) - c_1(t)(\dot{z}(t) - \dot{z}_l(t)) - Mg \\ m\ddot{z}_t(t) = +k_2(z_l(t) - z_t(t) - \Delta_{s2}) + c_2(t)(\dot{z}_l(t) - \dot{z}_t(t)) + \\ \quad -k_t(z_t(t) - z_r(t) - \Delta_t) - mg \\ \dot{c}_1(t) = -\beta c_1(t) + \beta c_{1,in}(t) \quad c_{min} \leq c_{1,in}(t) \leq c_{MAX} \\ \dot{c}_2(t) = -\beta c_2(t) + \beta c_{2,in}(t) \quad c_{min} \leq c_{2,in}(t) \leq c_{MAX} \\ L_{k1} = L_{c1} \\ L_{k2} = L_{c2} \end{cases} \quad (2)$$

The meaning of symbols used in (1) and (2) are represented in Fig.1. Particularly, consider that  $k_t$  and  $\Delta_t$  are stiffness and the unloaded deflection of the tire, respectively.  $c(t)$  and  $c_{in}(t)$  are the actual and requested damping of model (1), respectively. In the architectures (2)  $c_j(t)$  and  $c_{j,in}(t)$  stand for the actual and requested damping of the  $j$ -th shock absorber, respectively.  $\beta$  is the modulation bandwidth of the controllable shock-absorbers.  $(c_{min}, c_{MAX})$  is their controllability range. Considering model (1),  $k(t)$  and  $k_{in}(t)$  are the actual and requested spring coefficients, respectively.  $\gamma$  is the bandwidth of the controllable spring. The unloaded elongation is given by  $\Delta_s$  which can assume two possible values according to  $k_{in} \in \{k_{min}, k_{MAX}\}$ .  $(k_{min}, k_{max})$  is the controllability range of the spring.  $k_i$  and  $\Delta_i$  are the stiffness coefficient and the unloaded deflection of the  $i$ -th spring in (2).  $L_{c,i}$  and  $L_{k,i}$  are to the elongation of the  $i$ -th damper and  $i$ -th spring, respectively, in (2).

For simulation purposes the following parameters values are used throughout the paper.  $M = 400$  kg,  $m = 50$  kg,  $k_t = 250.000$  N/m,  $k_{min} = 5,000$  N/m,  $k_{MAX} = 40,000$  N/m,  $k_l = 5,700$  N/m,  $k_2 = 40,000$  N/m,  $c_{min} = 150$  Ns/m,  $c_{MAX} = 3,900$  Ns/m;  $\gamma = \beta = 40 \cdot 2\pi$ ;  $c_{nom} = 1,500$  Ns/m  $k_{nom} = 20,000$  N/m ([18], [21]).

Model (1) and model (2) are non-linear dynamical systems. And they were been compared and analyzed in [19]. Some conclusions are here reported:

- Model (1) is ideal and it may switch among two different springs, so in principle two equilibrium points are possible. However, with an appropriate choice of the unloaded lengths ( $\Delta_s = \Delta_{min}$  for  $k_{min}$  and  $\Delta_s = \Delta_{max}$  for  $k_{MAX}$ ), it is possible to obtain a unique equilibrium point regardless of the stiffness value  $k_{min}$  and  $k_{max}$ . Global uniform stability of Model (1) can be concluded assuming an ideal switching of both the stiffness and the damping and by solving the related LMI for the computation of a common Lyapunov function- (see [10] and references therein).
- The controllable devices in model (2) are shock absorbers, which have no influence on the equilibrium point. Due to the presence of only variable damping, Model (2) is strictly passive, hence the equilibrium is stable and robust with respect with any control law of damping and with any value of the system parameters

(see e.g. [7]).

As a first step, for control design purposes, it is interesting to understand if the two systems (1) and (2) are comparable in their controllability range, that is, in correspondence of the boundary values of the coefficients of controllable spring (as for model (1)) and of controllable dampers (as for model (2)). It is easy to see that: if the damping coefficient  $c_1(t) \rightarrow \infty$ , then the suspension system (2) is reduced to an ideal quarter car equipped with a classical semi-active suspension composed by spring  $k_2$  and the variable damper  $c_2(t)$ . Similarly, if the value of  $c_1(t)$  is comparable to  $c_2(t)$ , then the resulting suspension can be considered as an ideal quarter-car equipped with an equivalent spring given by the series of  $k_1$  and  $k_2$ .

IDEAL SYSTEM (IS)	DOUBLE SUSPENSION SYSTEM (DSS)
$k = k_{min} ; C = C_{min}$	$C_1 = C_2 = C_{min}$
$k = k_{MAX} ; C = C_{MAX}$	$C_1 \gg C_2 ; C_2 = C_{MAX}$

Table 1. Parameters settings for the dynamical equivalence between the three suspensions systems.

With reference to the settings in Table 1, Fig.2 shows the bode magnitude plots of the *comfort* transfer functions (from road profile  $z_r(t)$  to the body acceleration  $\ddot{z}(t)$ ) for the systems (1) and (2) (for computation see e.g. [17]). Notice that the bode plots are nearly indiscernible. This confirms that the three suspension architectures provide the same results in correspondence of the boundary of controllability range.

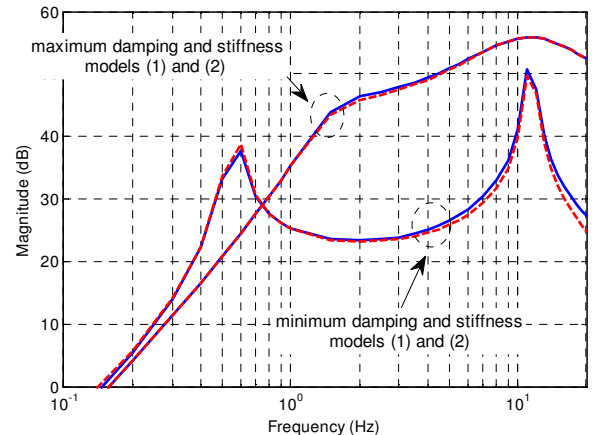


Figure 2. Comparison of the bode magnitude plots in extreme configurations of the two suspension systems (Table 1). Model (1): blue solid line. Model (2): red dotted line

### III. ANALYSIS OF SUSPENSIONS WITH VARIABLE-DAMPING-AND-STIFFNESS

In this section the following two main issues will be considered. What is the influence of the (fixed) spring

stiffness, in a classical variable-damping semi-active suspension? Assuming fast-switching damping and stiffness, what is the best possible performance achievable?

A. Sensitivity to stiffness in a variable-damping semi-active suspension.

In order to analyze how the introduction of the stroke limits affects the overall performances, it is necessary to embed a modified model of the spring stiffness, and to define a simulation test for the performance evaluation. An effective way of modeling the stroke limits (end-stops) of a suspension is to redefine the equations on the suspension spring  $k(t)$  as follows (with reference to equations (1)):

$$\begin{aligned} \dot{k}(t) &= -\beta k(t) + \beta k_{in}(t) \text{ if } |z - z_t - \Delta_s| < \Lambda \\ k(t) &= K \text{ if } |z - z_t - \Delta_s| \geq \Lambda; K \gg k_{max} \\ k_{in}(t) &\in [k_{min}, k_{max}] \end{aligned} \quad (3)$$

Symbols in (3) have the same meaning as the symbols of model (1). Moreover,  $\Lambda$  is the available stroke;  $K$  represents an “equivalent stiffness” of the end-stop zone; it is much higher than  $k_{max}$  (it is the typical stiffness of rubber bushings).

In order to highlight the trade-off arising from the introduction of the end-stops, the following evaluation test is defined:

- A standard road profile  $\bar{z}_r(t)$  is designed as an integrated white noise (random walk), band-limited within the frequency range [0-30]Hz. Its maximum amplitude is  $\bar{A} = 0.07m$ . Notice that this kind of signal resembles a realistic mild off-road profile (see [14]).
- During the simulation, the damping is controlled using the Mixed SH-ADD algorithm (which is proven to provide the quasi-optimal performance in terms of comfort for a genuine semi-active suspension system [17]). The variable-damping semi-active suspension system is compared in the range  $(k_{min}, k_{max})$ . The simulation test is repeated with and without end-stops ( $\Lambda = 0.2m$  is used, which represents the typical available stroke for a suspension of a vehicle designed for mild off-road conditions).

The effect of the introduction of the end-stops is displayed in Fig.3, where the body accelerations  $\ddot{z}(t)$  and the suspension travel  $z(t) - z_t(t)$  are plotted. Clearly, reaching the end-stop causes a dramatic deterioration of the comfort performances.

In order to better assess the suspension performances, the following indices can be taken into account:

$$J = \sqrt{\frac{\int_0^T (\ddot{z}(t))^2 dt}{\int_0^T (\bar{z}_r(t))^2 dt}} \quad (4)$$

$$J_{el} = \max|z - z_t| \quad (5)$$

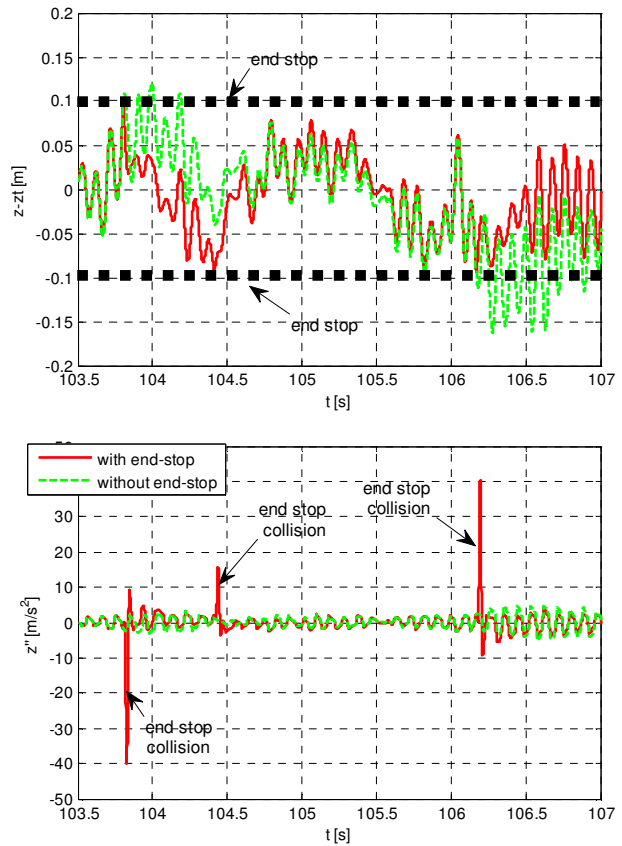


Fig.3. Time history of the suspension stroke (top) and the body acceleration (bottom), with and without the end-stops.

Both indices are computed assuming the standard road profile  $\bar{z}_r(t)$  as input. Notice that (4) is commonly used as comfort index for evaluating the suspension system performances ([6], [21]), and it represents a concise measure of the overall body motion. Index (5) is very simple and provides a measure of the required stroke travel for a particular working condition.

Considering a suspension with controllable damping and limited stroke travel, it is interesting to understand the potential compromises related with the design of spring stiffness. Notice that with damping control available the choice of the spring coefficient can be driven by considerations that differs from those for the classical suspension design. For this purpose, the IS model (with damping controlled by rule (3)) has been fed by  $\bar{z}_r(t)$ , and simulated with several levels of stiffness ( $k \in [k_{min}; k_{max}]$  with a 1,000N/m step). The results are summarized in Fig.4, where the performance indices (4) and (5) are displayed.

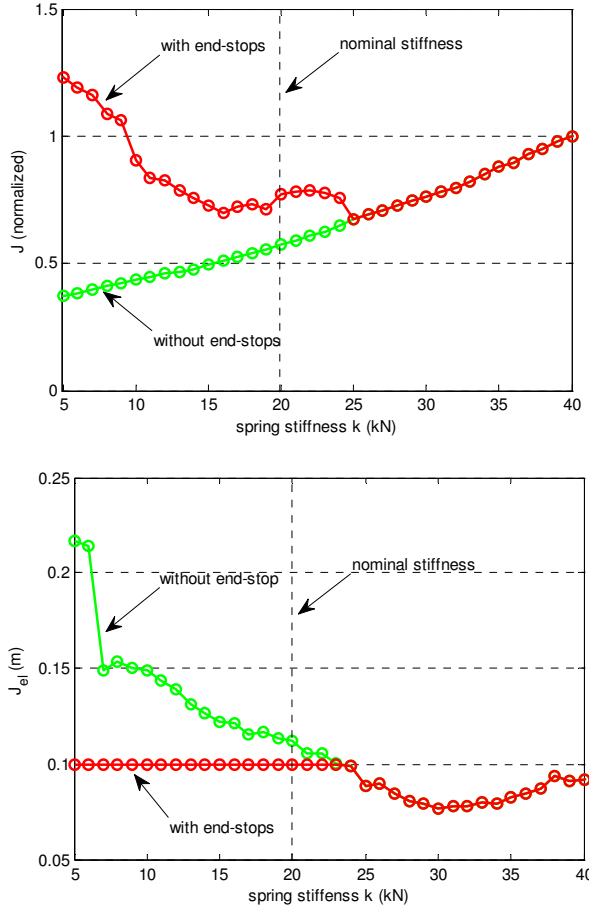


Fig.4. Disturbance transmission (top) and maximum suspension elongation (bottom) for suspensions with controlled damping, with respect to the spring stiffness ( $k \in [k_{min}, k_{MAX}]$ ). Without end-stops (green line); with end-stops (red line).

The analysis of Fig.4 clearly shows that:

- The dependence of the results with respect to the spring coefficient  $k$  is extremely non linear;
- The best performances in terms of comfort are guaranteed by a suspension with unlimited stroke travel.
- When the end-stops are included a compromise arises in the suspension system. In terms of body dynamics, the best spring stiffness is the softer that is simultaneously able to avoid the end-stop (in Fig.4 this is  $k = 25KN/m$ ). It would be interesting to understand if an appropriate control of stiffness may manage the stroke travel in a better way and provide good comfort without hard stops.
- 

#### B. Optimal predictive control: the benchmark

This analysis is worked out in a “ideal” setting: the IS model (1) is considered; perfect knowledge (also in the future) of the road disturbance is assumed; and no limits on the computational complexity are given.

Consider an ideal semi-active suspension with variable-damping-and-stiffness and limited stroke travel, as described by equations (1) and (3). Consider that the parameters of the

controllable shock-absorber and spring ( $c_{min}, c_{MAX}, k_{min}, k_{MAX}, \beta$  and  $\gamma$ ) are fixed. Moreover assume that  $c_{in}(t) = \{c_{min}, c_{MAX}\}$  and  $k_{in}(t) = \{k_{min}, k_{MAX}\}$  (two-state damper and two-state spring), so that the control action must select, at every sampling time, one out of four possible damping-stiffness combinations:

$$(c_{in}(t), k_{in}(t)) = \{(c_{min}, k_{min}); (c_{min}, k_{max}); (c_{max}, k_{min}); (c_{max}, k_{max})\}$$

The design problem of the control algorithm can be reduced to the following: consider a time window  $[0, T]$ , fixed initial conditions, and a given road profile  $\tilde{z}_r(t)$ ,  $t \in [0, T]$ ; consider the global performance index (4), find the sequence of digital control inputs  $(c_{in}(1 \cdot \Delta T), k_{in}(1 \cdot \Delta T)), (c_{in}(2 \cdot \Delta T), k_{in}(2 \cdot \Delta T)) \dots (c_{in}(k \cdot \Delta T), k_{in}(k \cdot \Delta T)) \dots (c_{in}(H \cdot \Delta T), k_{in}(H \cdot \Delta T))$ ,  $H = T/\Delta T$ , which minimizes  $J^*$ . The solution of the above control problem provides the best possible control strategy for the suspension, for that road profile.

In practice, it is impossible to implement in real-time such a globally-optimal control strategy on a real system. Even if an *a priori* knowledge of  $\tilde{z}_r(t)$ ,  $t \in [0, T]$  is assumed, the optimization task is formidable: the optimal solution must be searched among  $4^H$  possible sequences of digital control inputs (if  $T = 10$  s and  $\Delta T = 10$  ms, this means that  $H = 1000$ , which makes the optimization task almost impossible to be dealt with). Even if this problem cannot be solved in a real-time implementation, it can be solved numerically offline.

The optimization task hence is tough but tractable, and the performance index  $J^*$  of the *quasi*-optimal-predictive-control strategy can be computed offline for increasing values of scaling factor  $\alpha$  of the input signal. The results of this numerical analysis are condensed in Fig.5. They are compared to Mixed SH-ADD on a traditional variable-damping semi-active architecture. The results condensed in Fig.5 are very interesting, and can be easily interpreted:

- When the road disturbance amplitude is very small (hence it is unlikely to hit the end-stop) the best performances are achieved with a suspension equipped with a soft spring ( $k_{min}$ ) and with a controllable damper ruled by Mixed SH-ADD algorithm.
- When the road disturbance amplitude is very large, the suspension equipped with a hard spring ( $k_{max}$ ) and with a damper controlled by Mixed-SH-ADD provides the best performances in terms of overall comfort.
- For medium amplitude of road displacement, an appropriate stiffness control (*quasi*-optimal- predictive control) is able to overcome this trade-off and to ensure the best comfort. The optimal filtering capability cannot be achieved by any traditional variable-damping suspension system with fixed spring stiffness.

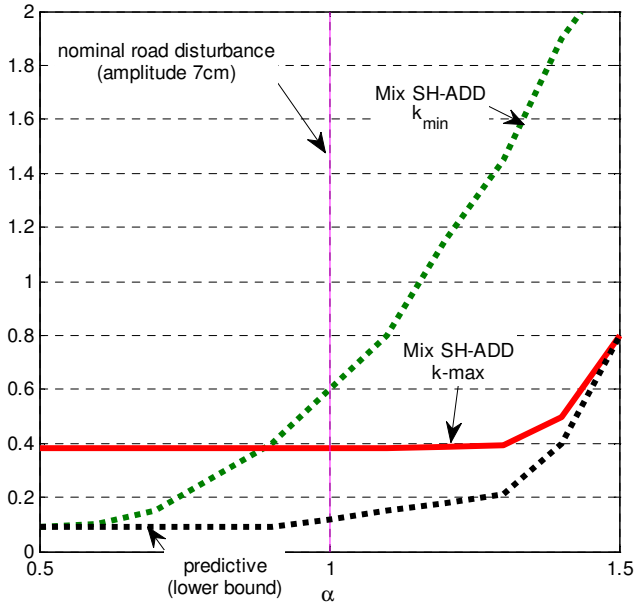


Fig.5. Comparison of the filtering performances of MIXED SH-ADD with low spring stiffness, high spring stiffness, and the numerically-computed quasi-optimal lower bound, as a function of the road input scaling factor  $\alpha$ .

#### IV. CONTROL STRATEGY: THE SSTSC ALGORITHM

This paper proposes a simple and innovative control algorithm named *Stroke-Speed-Threshold-Stiffness-Control (SSTSC)*.

This algorithm consists in a dynamic regulation of the suspension stiffness coefficient based on two thresholds (the first acts on the elongation  $|z - z_t|$ , the second on the elongation-speed  $|\dot{z} - \dot{z}_t|$  of the suspension), and on a sign-comparison of elongation and elongation-speed (2-dimensional control). With reference to the symbols used in IS (model (1)), the control law is the following:

$$\begin{aligned}
 (k_{in}(t) = k_{min}) \text{ AND } (c_{in}(t) = \text{Mixed SH-ADD}) \text{ if} \\
 (|z - z_t| \leq t_e) \text{ AND } ((|\dot{z} - \dot{z}_t| \leq t_{es}) \text{ OR } ((z - z_t) \cdot (\dot{z} - \dot{z}_t) \leq 0)) \\
 (6) \\
 (k_{in}(t) = k_{MAX}) \text{ AND } (c_{in}(t) = \text{Mixed SH-ADD}) \text{ otherwise}
 \end{aligned}$$

where  $t_e$  and  $t_{es}$  are the thresholds on the elongation and elongation speed, respectively; these two parameters are the two “tuning knobs” (to be optimized). Roughly speaking, when the suspension approaches the end-stop or when elongation and elongation-speed have the same sign and elongation-speed exceeds a specified threshold, the algorithm selects a high virtual stiffness coefficient in order to reduce elongation and to avoid suspension hard stops. At the same time, the damping of the suspension is regulated according to the Mixed SH-ADD rationale.

The SSTSC strategy is designed with reference to the IS (model (1)); however it can be implemented in both the suspension architecture (2) (DSS), according to the framework presented in Fig.6: the actual system is defined by equations (2), complemented with end-stop conditions (3); the control structure monitors measurable states of the

system (the body acceleration  $\ddot{z}(t)$  and its integrated signals; the suspension elongation  $z(t) - z_t(t)$  and its derivative); according to (6) the algorithm selects the damping and stiffness coefficients  $c_{in}(t)$  and  $k_{in}(t)$ , which are mapped into the equivalent damping of DSS.

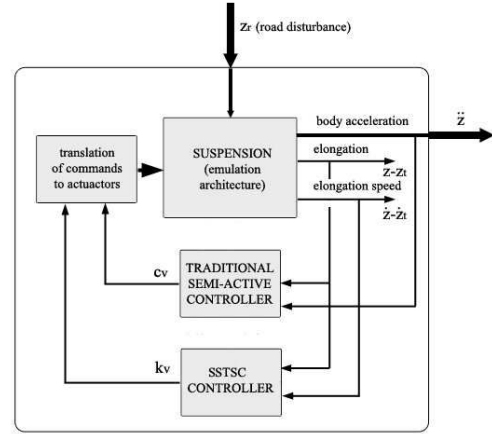


Fig.6. Implementation scheme of the SSTSC algorithm.

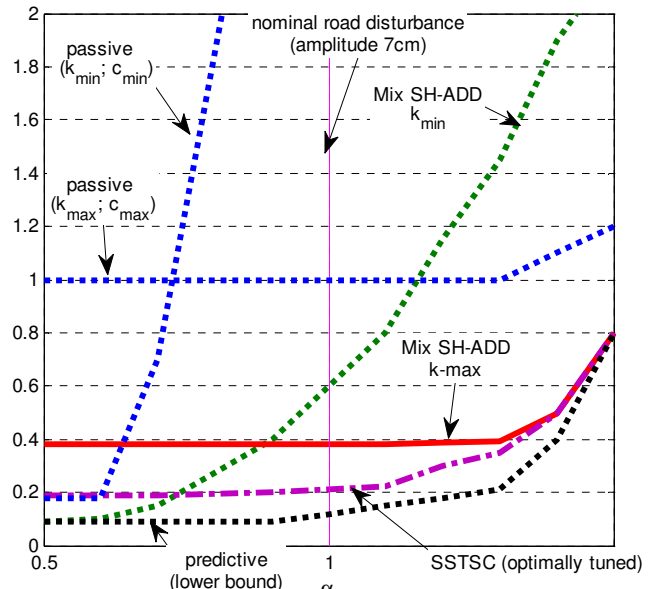


Fig.7. Analysis of the performance of different suspension configurations and algorithms, as a function of the input scaling factor  $\alpha$ .

To assess the performances of the SSTSC applied to the DSS architecture, the simulation activity (again, using the random signal with the scaling factor  $\alpha$ ) has been repeated in different configurations. The results are displayed in Fig.7, where the index  $J^*$  is displayed, as a function of the input scaling factor  $\alpha$ . The following remarks can be done:

- The SSTSC algorithm significantly outperforms the fixed-stiffness variable-damping semi-active suspension systems: this clearly shows the potential benefit of a variable-damping-and-stiffness suspension system.
- The SSTSC algorithm does not reach the lower-bound, computed with the quasi-optimal predictive control strategy; however the loss of performance with respect to

the “ideal” strategy is comparatively small.

- By focusing on the three traditional passive configurations, notice that the spread of performance is very large, and very dependent on the road input amplitude. This clearly shows that, in a traditional suspension, the problem of finding the correct compromise to tune the fixed values of damping and stiffness is a non trivial task.

- By inspecting the two variable-damping semi-active suspension, it is apparent that, thanks to the variable-damping capability (if coupled with a wise algorithm), the spread of performance is significantly reduced, when compared with the passive suspension. However, the choice of the fixed value of stiffness is still a non-trivial issue, which has not a unique solution.

## V. CONCLUSIONS

Starting from a suspension with controllable damper, this paper has studied how in a semi-active framework a control of the stiffness may improve the suspension performances. For this purpose a suspension architecture (DSS), capable of implementing “in practice” a variable-damping-and-stiffness suspension, have been proposed and discussed. This work has shown the remarkable potential benefits of a variable-damping-and-stiffness suspension architecture, obtained with practically implementable suspension system, and a simple but effective control algorithm (SSTSC). The research on this topic is now being developed along two mainstreams: the set-up of a real suspension system, for the experimental validation of the theoretical and simulation-based results obtained in this work, and the further development of the control algorithm.

## REFERENCES

- [1] Ahmadian M., B.A. Reichert, X. Song (2001). System Nonlinearities Induced by Skyhook Dampers. *Shock and Vibration*, vol.8, n.2, pp.95-104.
- [2] Fischer D., R. Isermann (2003). Mechatronic semi-active and active vehicle suspensions. *Control Engineering Practice*, vol.12, n.11, pp.1353-1367.
- [3] Giliomee, C. L. and S., Els P (1998). Semi-active hydro pneumatic spring and damper system. *Journal of Terramechanics*.
- [4] Goodall R.M., W. Kortüm (2002). Mechatronic developments for railway vehicles of the future. *Control Engineering Practice*, Vol.10, pp.887-898.
- [5] Hrovat, D. (1997). Survey of Advanced Suspension Developments and Related Optimal Control Applications. *Automatica*, Vol.33, n.10, pp. 1781-1817.
- [6] Isermann R. (2003). *Mechatronic Systems: Fundamentals*. Springer Verlag, UK.
- [7] Kahlil K. H (2002). *Nonlinear Systems*. Third Edition. Prentice Hall.
- [8] Karnopp, D. C., M.J. Cosby (1974). System for Controlling the Transmission of Energy Between Spaced Members. U.S. Patent 3,807,678.
- [9] Kiencke U., Nielsen L. (2000). *Automotive Control Systems for Engine, Driveline, and Vehicle*. Springer Verlag.
- [10] Liberzon, D (2003). *Switching in Systems and Control*. Birkhauser.
- [11] Liu Y, H. Matsuhisaa, H. Utsunoa (2008), “Semi-Active Vibration Isolation Sytem with Variable Stifness and Damping Control”. *Journal of Sound and Vibration*. Vol 313, Issue 1-2. Pasge 16-28
- [12] Nieto, A. J., et al. An analytical model of pneumatic suspensions based on an experimental characterization . *Journal of Sound and*

- Vibration* Volume 313, Issues 1-2, 3 June 2008, Pages 290-307 . 2007.
- [13] Poussot-Vassal C., O. Sename, L. Dugard, P. Gaspar, Z. Szabo and J. Bokor“A New Semi-active Suspension Control Strategy Through LPV Technique” In *Control Engineering Practice* 16(12), December, 2008, pp. 1519-1534.
- [14] Robson, J.D, C.J. Dodds (1970). *The Response of Vehicle Component to Random Road-Surface Undulations*. 13th FISITA congress, Brussels, Belgium.
- [15] Sammier D., Sename O., Dugard L. (2003). Skyhook and H $\infty$  control of semi-active suspensions: some practical aspects. *Vehicle System Dynamics*, Vol.39, n.4, pp. 279-308.
- [16] Savaresi S.M., E. Silani, S. Bittanti (2005). Acceleration-driven-damper (ADD): an optimal control algorithm for comfort-oriented semi-active suspensions. *ASME Transactions: Journal of Dynamic Systems, Measurement and Control*, vol.127, n.2, pp.218-229.
- [17] Savaresi S.M., C. Spelta (2007). Mixed Sky-Hook and ADD: Approaching the Filtering Limits of a Semi-Active Suspension. *ASME transactions: Journal of Dynamic Systems, Measurement and Control*. Vol. 129, Issue 4, pp. 382-392.
- [18] Savaresi S.M., C. Spelta (2009): A Single-Sensor Control Strategy for Semi-Active Suspensions. *IEEE Transaction on Control System Technology*. Vol. 17, No. 1, pp 143-152.
- [19] Spelta C., Cutini M., Bertinotti S.A., Savaresi S.M., Previdi F., Bisaglia C and Bolzern P. (2009). “A New Concept of Semi-Active Suspension with Controllable Damper and Spring”. To appear in the *Proceeding of the European Control Conference 2009*, 23-26 August 2009, Budapest, Hungary.
- [20] Spelta C., F. Previdi, S.M. Savaresi, G. Fraternali, N. Gaudio (2009). “Control of Magnetorheological Dampers for Vibration Reduction in a Washing Machine”. *Mechatronics*. Vol. 19, pp 410-421
- [21] Williams R.A. (1997). *Automotive Active Suspensions*. IMechE, Vol. 211, Part D, pp. 415-444.