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Optical feedback sustained self-pulsations in semiconductor lasers (*)

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Summary. — In this work we study the dynamics of a single-cavity semiconductor laser device, able to work in self-pulsating regime as a consequence of a nonlinearity distributed in the whole active region and driven by the electromagnetic-field intensity in the laser cavity. Numerical values of the nonlinearity degree necessary to establish the self-pulsating regime are given which can be used for a comparison with experimental measurements of semiconductor materials properties. The optical feedback influence on the self-pulsating device is also studied showing how pulsation characteristics are affected by the presence of a reflected signal and pointing out the feedback ability to cause their generation or quenching. The possibility of using current and optical feedback for the pulsation characteristics control and for the switch of the laser output is discussed.

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1. – Introduction

Self-pulsating behaviour of semiconductor lasers is usually related to the presence of saturable absorbing centers in the active material [1]. These ones are normally assumed to be separated from the active region and the pulsation generation is considered as the consequence of the interplay between the dynamics of the active and the absorbing section [2].

In this work a single-cavity device is studied, which can operate in the self-pulsating regime owing to the presence, in the whole active region, of a nonlinearity dependent on the electromagnetic-field intensity in the laser cavity, responsible for a saturable absorption mechanism in the active material. The assumption that the absorption mechanism is at work distributively in the whole active region simplifies the system dynamics, since there is no competition between different sections.

(*) The authors of this paper have agreed to not receive the proofs for correction.

The device dynamics is described through a rate equation model properly adjusted to take into account the nonlinearity effect. The model can therefore furnish the numerical values of the nonlinearity degree necessary to establish the self-pulsating regime: these ones can be compared with experimental measurements of the intrinsic properties of semiconductor materials.

Using the results obtained in previous works [3, 4], where a complete analysis of the feedback influence on semiconductor lasers performances was presented, optical feedback effects on the self-pulsating device are also investigated, showing how the feedback influences the pulsation characteristics and how it can be responsible for their appearance or disappearance.

2. – Theoretical basis of the self-pulsating device working

The assumption is made that the dynamics of a semiconductor stripe laser is influenced by the presence, in the whole active region, of a nonlinearity driven by the electromagnetic-field intensity in the laser cavity, *i.e.* by the photon number n_{ph} as $n_{\mathrm{ph}} \propto |E|^2$, where $|E|$ is the field amplitude. The nonlinearity causes an absorption mechanism in the material, which, when it becomes sufficiently strong, saturates causing the establishment of the self-pulsating regime.

The nonlinearity effect can be modelled, in a rate equation system, as a decrease of the electromagnetic-intensity relaxation coefficient $2k$, in the following way:

$$2k = \overline{2k}(1 - \eta n_{\mathrm{ph}})\,; \tag{1}$$

the term ηn_{ph} (η is a proper parameter controlling the nonlinearity amount) accounts for the nonlinearity presence, while $\overline{2k}$ is the usual relaxation coefficient.

The mechanism responsible for pulsation appearance can be related to intrinsic properties of semiconductor materials, as the ones predicted by the nonlinear optics theory. The third-order susceptibility $\chi^{(3)}$ [5, 6] typical of semiconductors can in fact be responsible for many effects. In particular, considering the optical Kerr effect, a relation can be established between the variation of the relaxation coefficient previously introduced and the one of the refractive index predicted by the Kerr effect (see appendix). In this way, the following expression for the parameter η can be obtained:

$$\eta = 80\pi \,\frac{h\nu}{V}\, n_2\,,$$

where h is the Planck constant, ν is the frequency of the emitted radiation, V is the volume of the active material and n_2 is the nonlinear refractive index. Numerical simulation results can then be compared with experimental measurements, even though a possible relation with other types of material nonlinearities different from the Kerr effect is not excluded.

3. – Rate equation model and numerical simulation results

Using the theory of a two-level system interacting with a single-mode electromagnetic field [7, 8] and modelling the nonlinearity effect as previously explained, the following couple of rate equations can be obtained, in which the dynamic variables are

TABLE I. – *Parameter values used in numerical simulations.*

Physical quantity	Symbol	Value
Simulated-emission coefficient	$4\chi^2/\gamma_\perp$	$10^4\ \mathrm{s}^{-1}$
Electromagnetic-field relaxation coefficient	$2k$	$(2\cdot 10^{-12}\ \mathrm{s})^{-1}$
Population inversion relaxation coefficient	$\gamma_\parallel$	$(2\cdot 10^{-9}\ \mathrm{s})^{-1}$
Spontaneous-emission coefficient	β	10^{-4}
Normalized injection current	I/I_{th}	1.5

the number of photons n_{ph} and the number of injected carriers n_{c}:

$$\frac{\mathrm{d}n_{\mathrm{ph}}}{\mathrm{d}t} = \frac{4\chi^2}{\gamma_\perp} n_{\mathrm{c}} n_{\mathrm{ph}} - 2k(1-\eta n_{\mathrm{ph}})\, n_{\mathrm{ph}} + \beta\gamma_\parallel n_{\mathrm{c}}\,,$$

$$\frac{\mathrm{d}n_{\mathrm{c}}}{\mathrm{d}t} = -\frac{4\chi^2}{\gamma_\perp} n_{\mathrm{c}} n_{\mathrm{ph}} - \gamma_\parallel n_{\mathrm{c}} + \frac{I}{e}\,.$$

The electromagnetic-field phase is neglected by means of the mean-field approximation since it does not change the main features of the model. $2k$, $\gamma_\parallel$ and $\gamma_\perp$ are the relaxation coefficients of the electromagnetic-field amplitude, population inversion and polarization, respectively; χ describes the coupling between the electromagnetic field and the active material and it is linked to the relative steepness of the gain *vs.* injection current characteristic; β is the spontaneous-emission coefficient.

The rate equation system has been solved using a fifth-order Runge-Kutta (Fehlberg modified) algorithm with the laser parameters listed in table I.

As expected, the model presents a Hopf bifurcation dependent on the value of the parameter η, *i.e.* on the nonlinearity amount. Figure 1 reports, for different injection

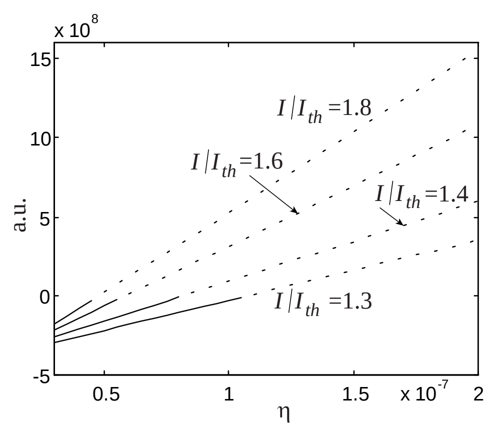

Fig. 1. – Eigenvalue real part of the linearized rate equation system against the parameter η at different injection current values (stable behaviour in continuous line, unstable behaviour in dashed line).

current values, the real part of the complex-coniugate pair of eigenvalues of the linearized rate equation system: it can be observed that the parameter η is responsible for the transition between damped and undamped oscillations.

In figs. 2 the different behaviours of photon dynamics below and above the bifurcation point can be observed: in the first case (a) the device operates in continuous regime, in the second one (b) it operates in self-pulsating regime.

In fig. 3 the boundary between the continuous and the self-pulsating regime as a function of the injection current can be observed. Higher currents, increasing the laser cavity optical field, enhance the nonlinearity effect requiring a lower η for the self-pulsating regime establishment. Current can then be used for controlling the switch of the laser output between the two dynamic regimes.

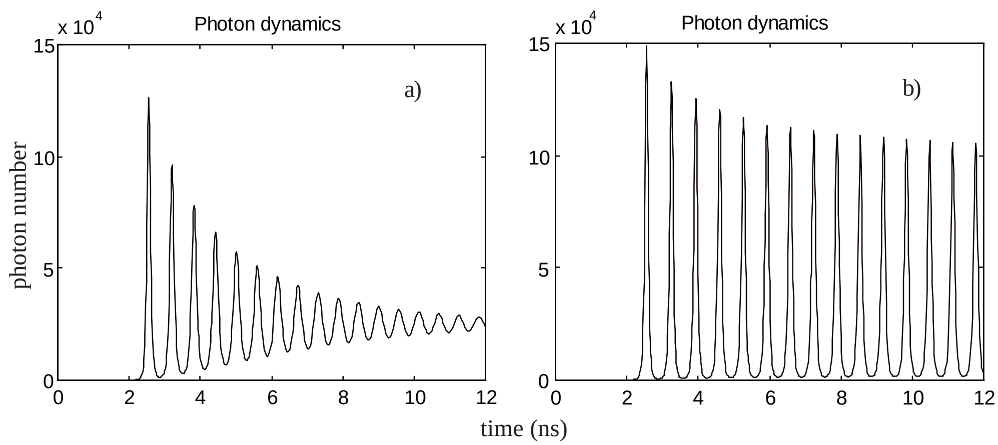

Fig. 2. – a) Photon dynamics with $\eta = 2\cdot 10^{-8}$; b) photon dynamics with $\eta = 8\cdot 10^{-8}$.

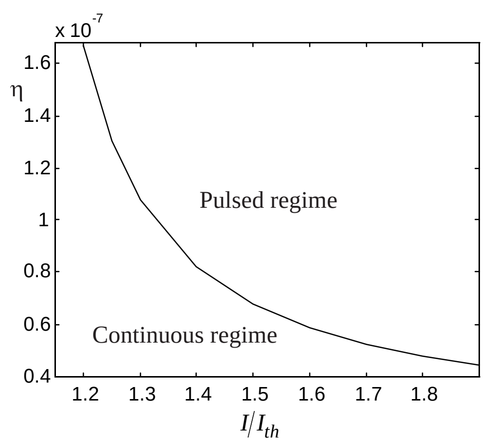

Fig. 3. – Boundary between continuous and pulsed regime as a function of the normalized injection current.

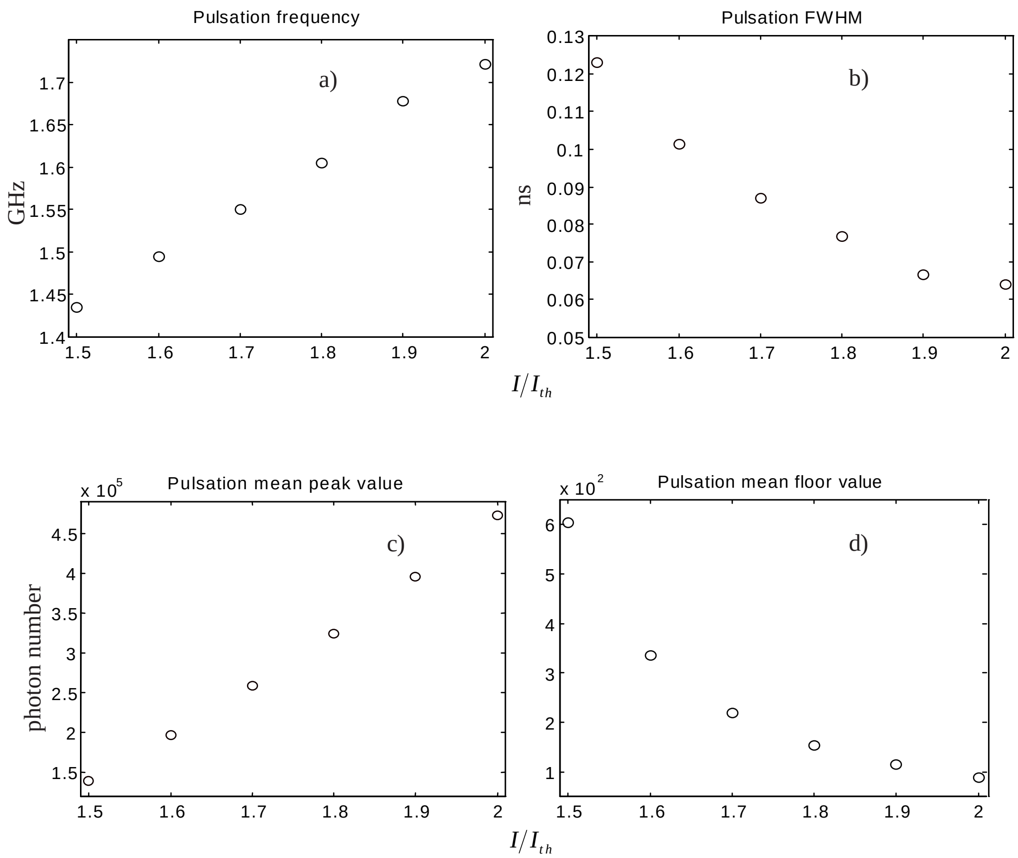

Fig. 4. – a) Pulsation frequency for different levels of normalized injection current ($\eta = 10^{-7}$); b) pulsation FWHM for different levels of normalized injection current ($\eta = 10^{-7}$). c) Pulsation mean peak value for different levels of normalized injection current ($\eta = 10^{-7}$); d) pulsation mean floor value for different levels of normalized injection current ($\eta = 10^{-7}$).

Figures 4 show the current influence on the pulsation characteristics: the obtained results point out the possibility of using the injection current for modelling them in frequency, width and amplitude.

4. – Optical feedback influence on the self-pulsating device dynamics

It is well known that the optical feedback can significantly affect the dynamic behaviour of a semiconductor laser [9-11]. In previous works [3, 4] the feedback influence on semiconductor laser devices was analysed, pointing out, on the basis of experimental results, that in the presence of a reflected signal the differential quantum efficiency η_d, corresponding to the slope of the characteristic curve, can increase or decrease in comparison to its value in the absence of feedback. This behaviour was properly modelled in a rate equation system.

Using the obtained results, optical feedback effects on the performances of the self-pulsating device can be studied extending the rate equation model in the following way:

$$\frac{\mathrm{d}n_{\mathrm{ph}}}{\mathrm{d}t} = \frac{4\chi^2}{\gamma_\perp} n_{\mathrm{c}} n_{\mathrm{ph}} - 2(k-k')(1-\eta n_{\mathrm{ph}})\, n_{\mathrm{ph}} + \beta\gamma_{\parallel} n_{\mathrm{c}} ,$$

$$\frac{\mathrm{d}n_{\mathrm{c}}}{\mathrm{d}t} = -\frac{4\chi^2}{\gamma_\perp} n_{\mathrm{c}} n_{\mathrm{ph}} - \gamma_{\parallel} n_{\mathrm{c}}(1+a n_{\mathrm{ph}}) + \frac{I}{e} .$$

Feedback is represented by the term $2k' n_{\mathrm{ph}}$ in the photon dynamics and the term $-\gamma_{\parallel} a n_{\mathrm{ph}} n_{\mathrm{c}}$ in the carrier dynamics (the parameters k' and a are both related to the feedback ratio f between reflected and transmitted power and experimentally tunable). The first one takes into account the amount of reinjected photons, while the second one represents the coherent interaction between reinjected photons and carriers. The behaviour of the differential quantum efficiency in the presence of a reflected signal is considered in the sign of the parameter a: when a is negative the differential quantum efficiency increases in comparison to its value in the absence of feedback, while, when a is positive, the differential quantum efficiency decreases.

Numerical simulations of the rate equation model obtained show how the optical-feedback presence affects both amplitude and period of the pulsations [12-14] and how it influences the bifurcation point, *i.e.* the nonlinearity amount necessary to induce the self-pulsation phenomena.

Figures 5 show that, when a is negative ($\eta_{\mathrm{d}}^{\mathrm{f}} > \eta_{\mathrm{d}}$), the amplitude of the pulsations increases: the pulsation mean peak value increases while the pulsation mean floor value decreases. On the other hand, when a is positive ($\eta_{\mathrm{d}}^{\mathrm{f}} < \eta_{\mathrm{d}}$) the pulses decrease in amplitude: the pulsation mean peak value decreases while the pulsation mean floor value increases.

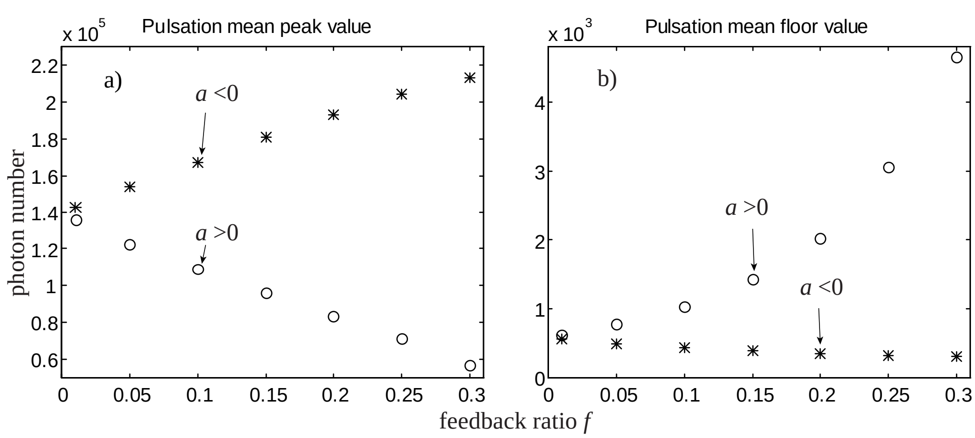

Fig. 5. – a) Pulsation mean peak value as a function of the feedback ratio f in the two cases $a<0$ and $a>0$ ($\eta = 10^{-7}$); b) pulsation mean floor value as a function of the feedback ratio f in the two cases $a<0$ and $a>0$ ($\eta = 10^{-7}$).

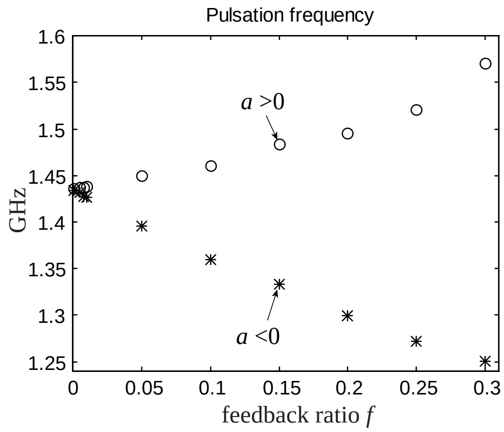

Fig. 6. – Pulsation frequency *vs.* feedback ratio f when $\eta = 10^{-7}$, in the two cases $a < 0$ and $a > 0$.

Figure 6 shows the pulsation frequency as a function of the feedback ratio f: when a is negative ($\eta_d^f > \eta_d$) the frequency lowers, while when a is positive ($\eta_d^f < \eta_d$) the frequency increases.

Finally, the data reported in table II, represented in fig. 7, show how the boundary between the continuous and the pulsed regime as a function of the injection current changes in the presence of optical feedback.

The obtained results about optical-feedback influence on the self-pulsating device dynamics perfectly agree with the nonlinear mechanism responsibile for pulsation appearance. When a is negative ($\eta_d^f > \eta_d$) the field intensity in the laser cavity increases enhancing the nonlinearity effect: a lower η can then establish the self-pulsating regime and a value of reflected power sufficiently strong can cause the laser output to switch from the continuous to the self-pulsating regime. On the other hand, when a is positive ($\eta_d^f < \eta_d$) the field intensity in the laser cavity decreases

TABLE II. – *Numerical values of the bifurcation point between continuous and pulsed regime for different values of the normalized injection current in the absence of feedback and in the presence of feedback, for the two cases $a < 0$ and $a > 0$.*

I/I_{th}	No feedback	With feedback ($f = 0.1$)	
		$a < 0$	$a > 0$
1.2	$16.7 \cdot 10^{-8}$	$15.39 \cdot 10^{-8}$	$19.37 \cdot 10^{-8}$
1.25	$13.04 \cdot 10^{-8}$	$12.09 \cdot 10^{-8}$	$15.08 \cdot 10^{-8}$
1.3	$10.8 \cdot 10^{-8}$	$10.05 \cdot 10^{-8}$	$12.46 \cdot 10^{-8}$
1.4	$8.19 \cdot 10^{-8}$	$7.69 \cdot 10^{-8}$	$9.45 \cdot 10^{-8}$
1.5	$6.76 \cdot 10^{-8}$	$6.37 \cdot 10^{-8}$	$7.79 \cdot 10^{-8}$
1.6	$5.86 \cdot 10^{-8}$	$5.54 \cdot 10^{-8}$	$6.75 \cdot 10^{-8}$
1.7	$5.24 \cdot 10^{-8}$	$4.97 \cdot 10^{-8}$	$6.03 \cdot 10^{-8}$
1.8	$4.79 \cdot 10^{-8}$	$4.55 \cdot 10^{-8}$	$5.51 \cdot 10^{-8}$
1.9	$4.45 \cdot 10^{-8}$	$4.23 \cdot 10^{-8}$	$5.12 \cdot 10^{-8}$
2	$4.18 \cdot 10^{-8}$	$3.98 \cdot 10^{-8}$	$4.81 \cdot 10^{-8}$

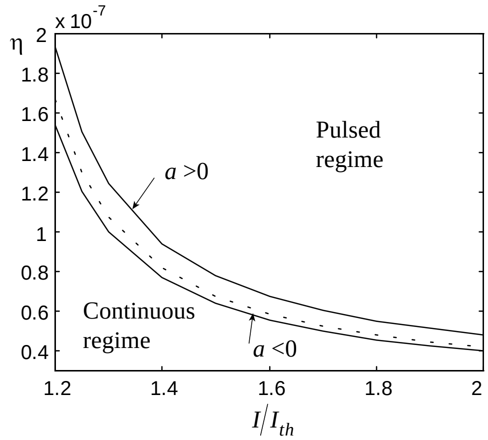

Fig. 7. – Boundary between continuous and pulsed regime as a function of the normalized injection current, without feedback (dashed line) and with feedback ratio $f = 0.1$, for the two cases $a < 0$ and $a > 0$.

attenuating the nonlinearity role: bifurcation then occurs at a higher η and for a value of reflected power sufficiently strong the nonlinear mechanism can become no more sufficient to sustain the pulsations, restablishing the continuous regime. Experimental results showed the optical-feedback ability to be responsible for pulsation generation or quenching [15].

On the basis of the obtained results, the optical feedback can then be used in two opposite ways for the nonlinearity and consequently for the device dynamics control.

5. – Conclusions

In this work we studied the dynamics of a single-cavity semiconductor laser device which can operate in self-pulsating regime owing to the presence, in the whole active region, of a nonlinearity which depends on the electromagnetic-field intensity in the laser cavity. The advantage of such device is that the absence of multiple cavities avoids the existence of chaotic behaviours due to the competition between different sections.

The device dynamics has been described through a rate equation model properly adjusted to take into account the nonlinearity effect, whose numerical solution gives the values of the nonlinearity degree necessary to induce the self-pulsation phenomena. These values can be compared with experimental measurements of semiconductor materials as the index variations predicted by the optical Kerr effect, without excluding a possible relation with other types of material optical non-linearities.

The injection current and the optical-feedback influence on the pulsation characteristics have been studied by pointing out their ability to model them and also to switch the laser output. Particularly, the possibility of using optical feedback for controlling the device dynamics in two different ways has been discussed.

APPENDIX

The index variation predicted by the optical Kerr effect is [5]

$$\Delta n = \frac{80\pi}{cn_0} n_2 I , \tag{A.1}$$

where n_2 is the nonlinear refractive index (e.s.u.), I is the intensity of the applied field, n_0 is the refractive index of the material and c is the light velocity. The energy density in the material can be expressed as

$$\varrho = \frac{I}{cn_0} = \frac{n_{\text{ph}} h\nu}{V} , \tag{A.2}$$

where n_{ph} is the photon number, h is the Planck constant, ν is the frequency of the emitted radiation and V is the volume of the active material. By introducing (A.2) in (A.1) and comparing the variation of the refractive index with the variation of the relaxation coefficient (1) describing the nonlinearity presence, that is

$$\Delta n = \frac{80\pi}{cn_0} n_2 \frac{n_{\text{ph}} h\nu}{V} cn_0 = \eta n_{\text{ph}},$$

the following expression for parameter η can be obtained:

$$\eta = 80\pi \frac{h\nu}{V} n_2 .$$

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