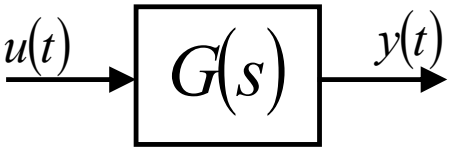
Lezione 10.
Risposta in frequenza

Schema della lezione

1. Risposta alla sinusoide
2. Teorema della risposta in frequenza
3. Risposta in frequenza

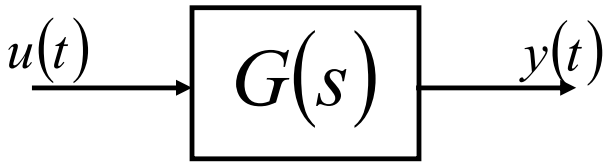
1. Risposta alla sinusoide



$$G(s) = \frac{\mu}{s^g} \frac{\prod_i (1 + sT_i)}{\prod_i (1 + s\tau_i)}$$ As. stabile

$$u(t) = A\sin(\omega t) \quad \Rightarrow \quad y(t) = ?$$

2. Teorema della risposta in frequenza



As. stabile

$$u(t) = A\sin(\bar{\omega}t)$$

Allora a transitorio esaurito (al lato pratico per $t > t_a$)

$$y(t) \cong B\sin(\bar{\omega}t + \varphi)$$

dove: $B = |G(j\bar{\omega})|A$

$$\varphi = \angle G(j\bar{\omega})$$

indipendentemente dalle condizioni iniziali del sistema

Esempio

$$G(s)=\frac{1}{1+s}$$

$$u_1(t)=\sin(0.5t)$$

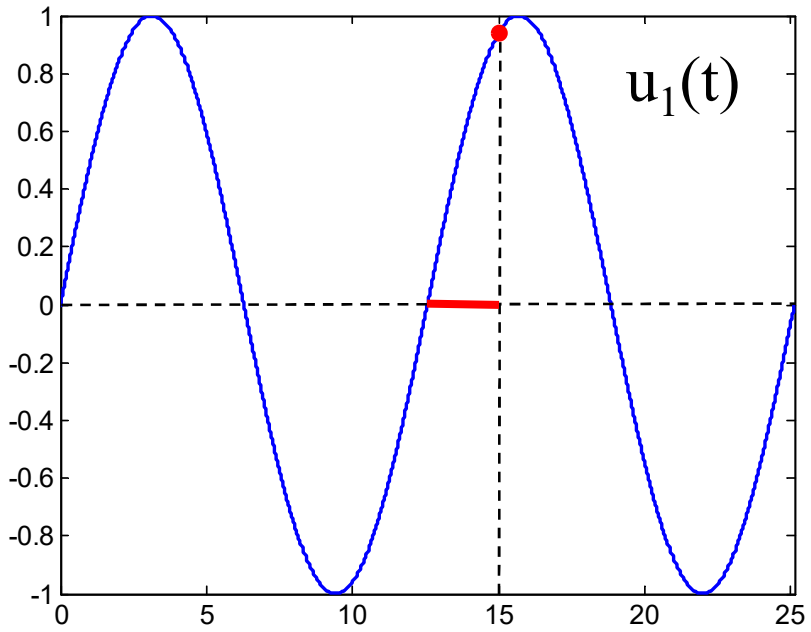
$$\omega_1=0.5\ \text{rad/s}$$

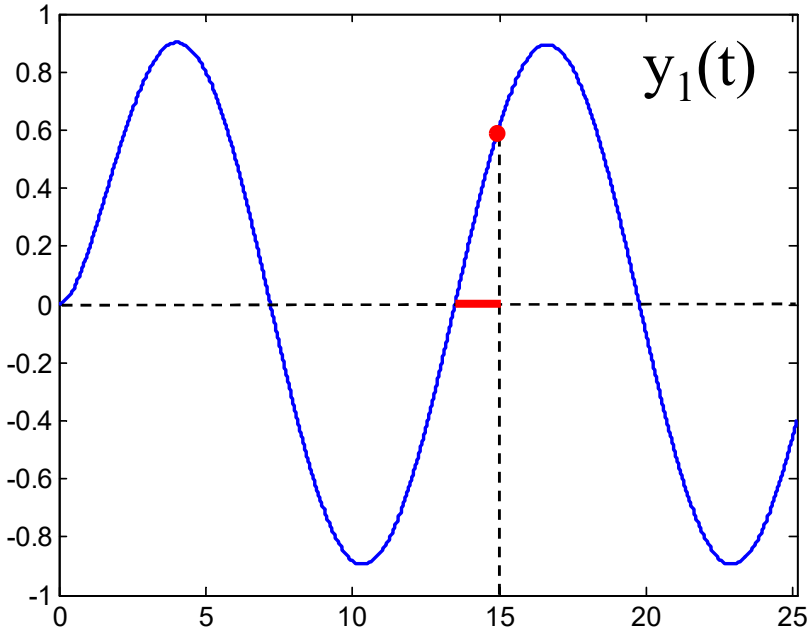
$$\Rightarrow \begin{cases} |G(j0.5)|=\dfrac{1}{|1+j0.5|}=0.89 \\ \angle G(j0.5)=-\text{atan}(0.5)=-26.6^\circ \end{cases}$$

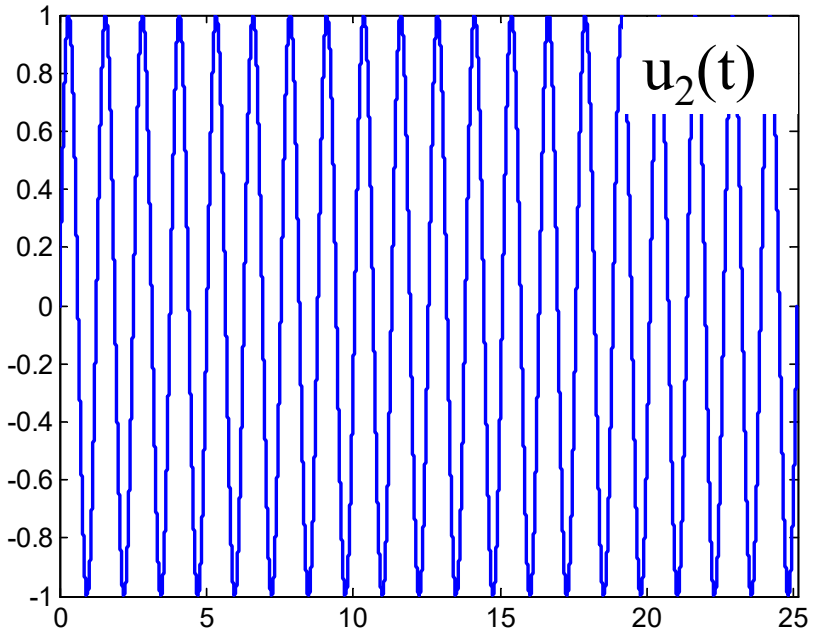
$$u_2(t)=\sin(5t)$$

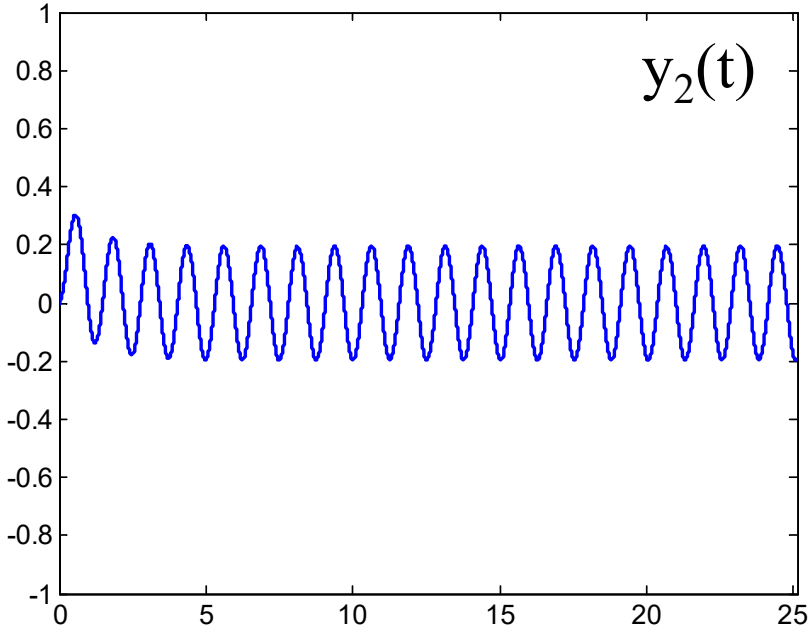
$$\omega_2=5\ \text{rad/s}$$

$$\Rightarrow \begin{cases} |G(j5)|=\dfrac{1}{|1+j5|}=0.20 \\ \angle G(j5)=-\text{atan}(5)=-78.6^\circ \end{cases}$$


$u_1(t)$


$y_1(t)$


$u_2(t)$


$y_2(t)$

3. Risposta in frequenza

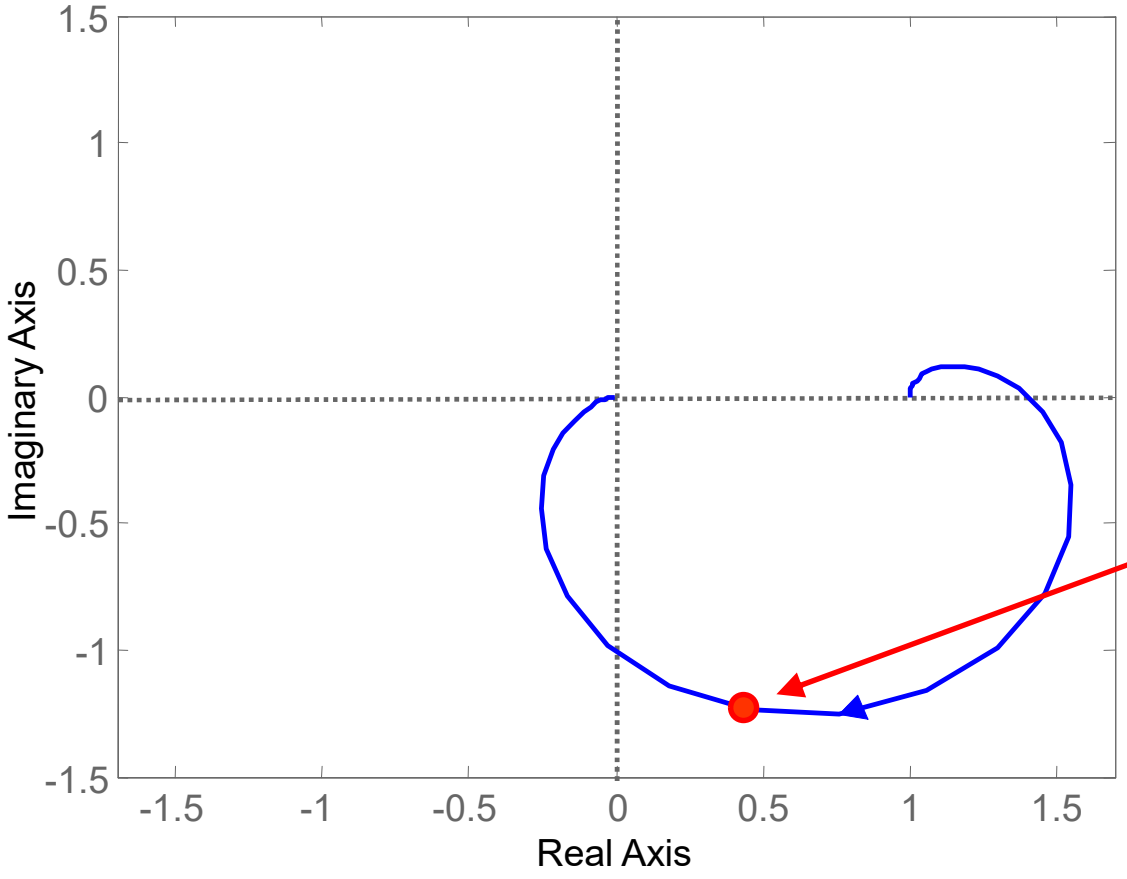
$$G(j\omega) \quad \text{per tutti gli} \quad \omega \geq 0$$

Funzione complessa di variabile reale

$$\omega \longrightarrow G(j\omega)$$

Esempio

$$G(s)=\frac{1+s}{\left(1+\frac{1}{3}s\right)\left(1+\frac{1}{5}s\right)^2} \Rightarrow G(j\omega)=\frac{1+j\omega}{\left(1+\frac{1}{3}j\omega\right)\left(1+\frac{1}{5}j\omega\right)^2}$$



$$G(j5)\cong 0.44-1.23\,j$$

$$|G(j5)|\cong\sqrt{0.44^2+1.23^2}=$$

$$=1.306$$

$$\arg G(j5)\cong \operatorname{atan}\frac{-1.23}{0.44}\cong$$

$$\cong -70°,3$$

4. Matlab

`[H,W] = freqresp(SYS)`

restituisce la risposta in frequenza calcolata in valori di pulsazione predefiniti W.