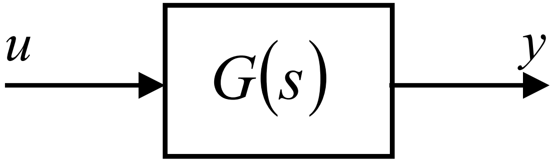
Lezione 9.
Schemi a blocchi

Schema della lezione

1. Elementi base di uno schema a blocchi
2. Esempio esplicativo
3. Regole di elaborazione
   serie, parallelo, retroazione
4. Stabilità
   serie, parallelo, retroazione

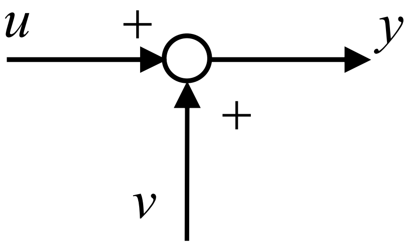
1. Elementi base di un schema a blocchi

blocco



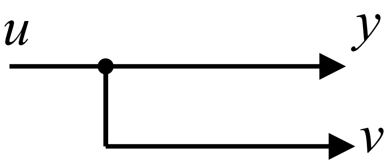
$$Y(s) = G(s)U(s)$$

nodo sommatore



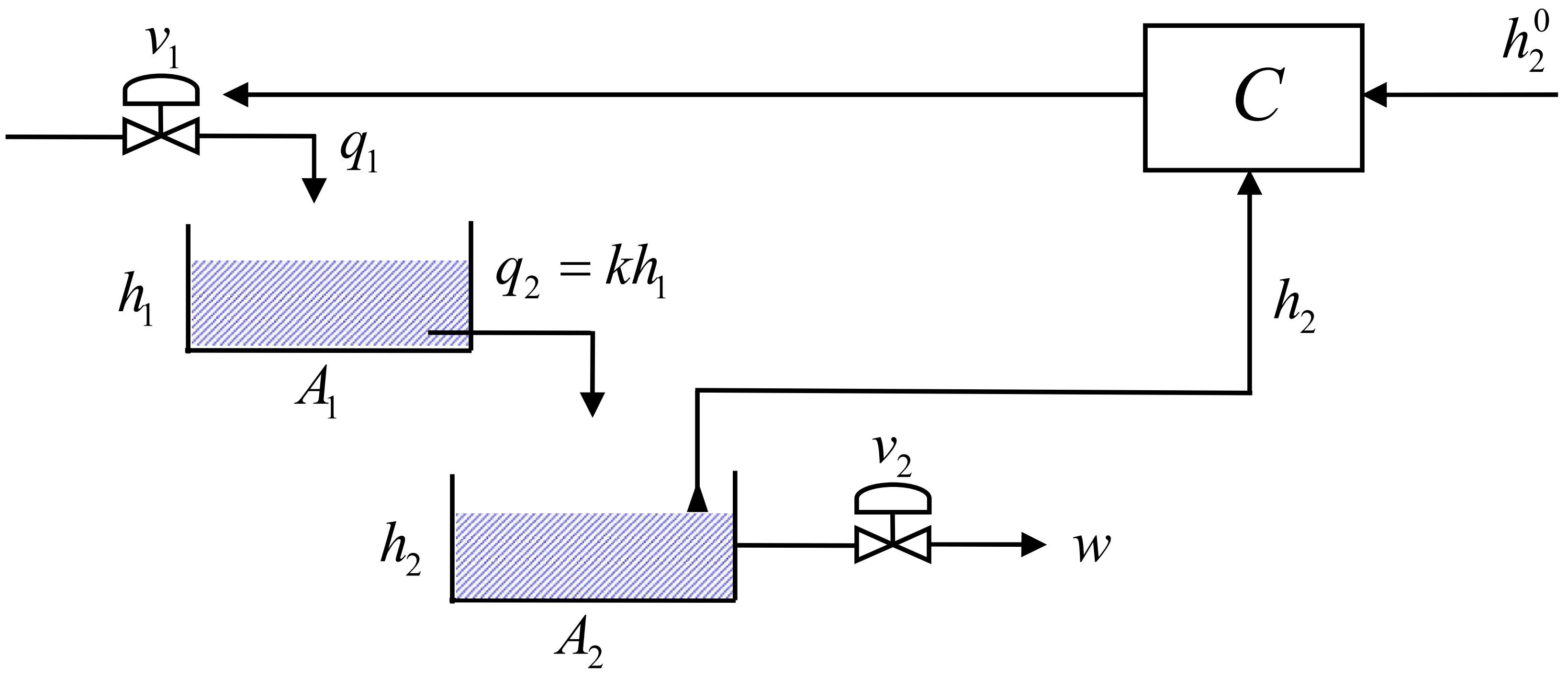
$$Y(s) = U(s) + V(s)$$

punto di diramazione



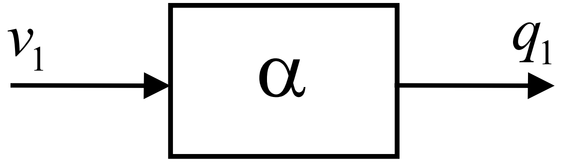
$$Y(s) = V(s) = U(s)$$

2. Esempio esplicativo



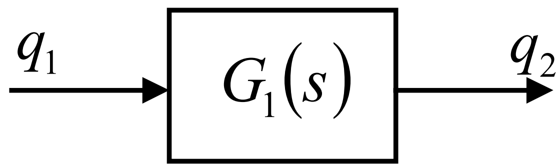
Valvola 1

$$q_1(t) = \alpha v_1(t)$$



Serbatoio 1

$$\begin{cases} A_1 \dot{h}_1(t) = q_1(t) - k h_1(t) \\ q_2(t) = k h_1(t) \end{cases}$$



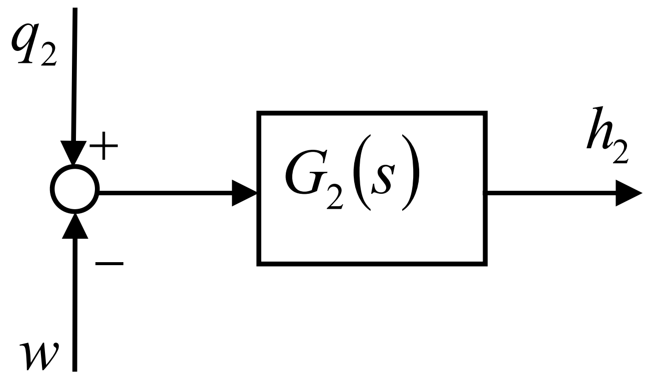
$$G_1(s) = \frac{k}{A_1 s + k}$$

Serbatoio 2

$$A_2\dot{h}_2(t) = q_2(t) - w(t)$$

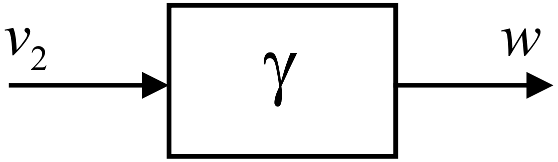
$$H_2(s) = \frac{1}{A_2 s}\left(Q_2(s) - W(s)\right)$$

$$\frac{1}{A_2 s} \rightarrow G_2(s)$$



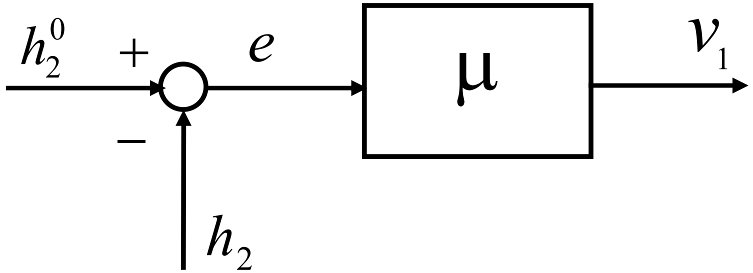
Valvola 2

$$w(t) = \gamma v_2(t)$$



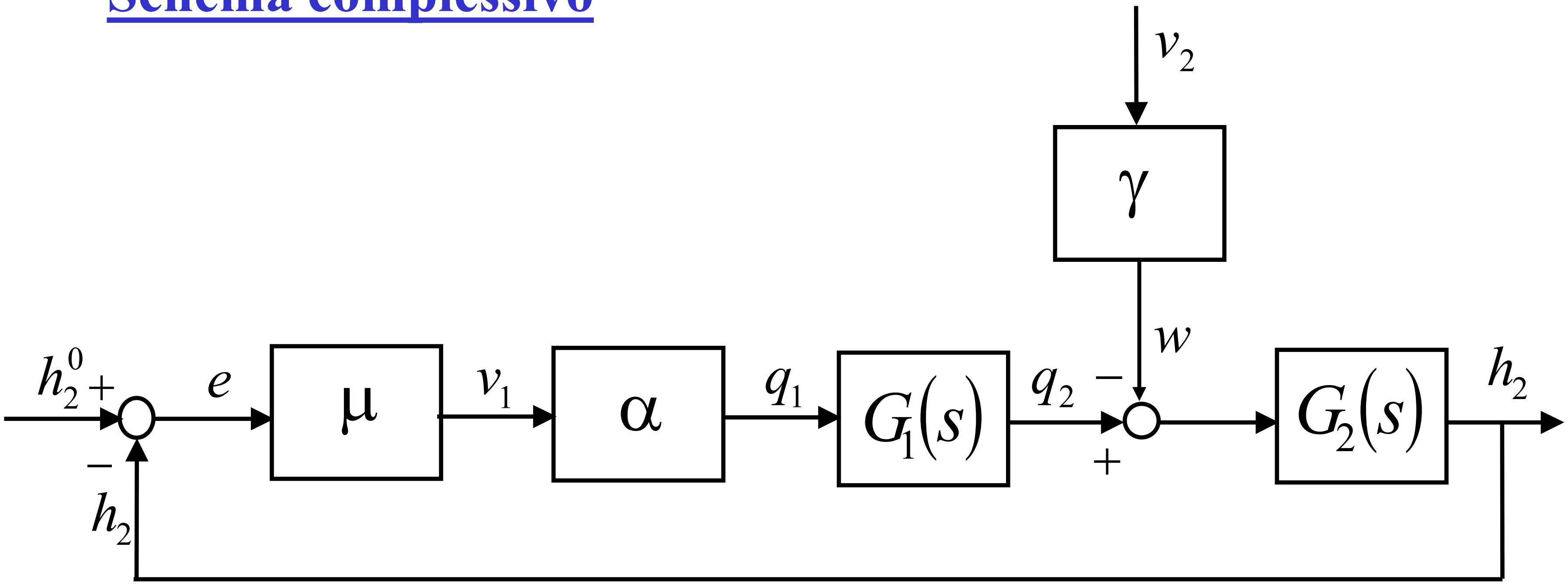
Controllore

$$v_1(t) = \mu \underbrace{\left(h_2^0(t) - h_2(t)\right)}_{e(t)}$$



Controllore proporzionale

Schema complessivo



FdT tra h_2^0 e h_2 ?

FdT tra v_2 e h_2 ?

FdT tra v_2 e e ?

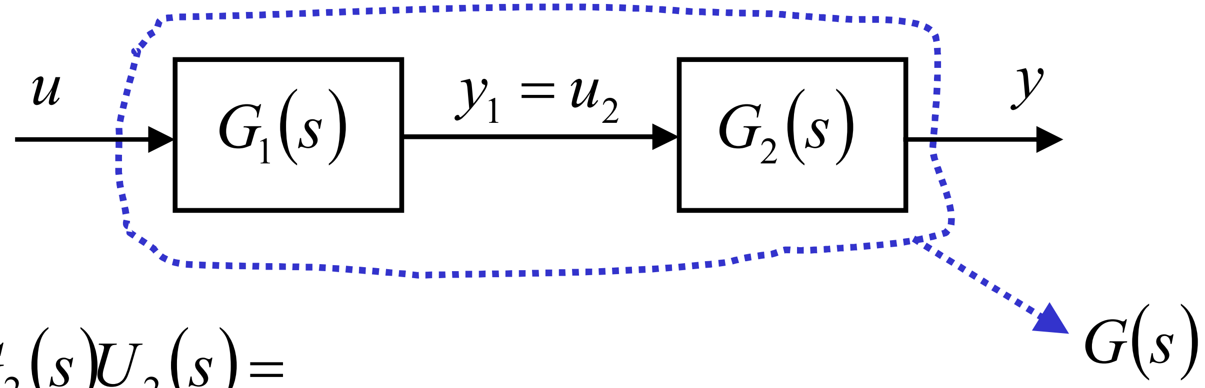
… altre FdT ?

3. Schemi a blocchi : regole di elaborazione

Blocchi

- in serie
- in parallelo
- in retroazione

Blocchi in serie



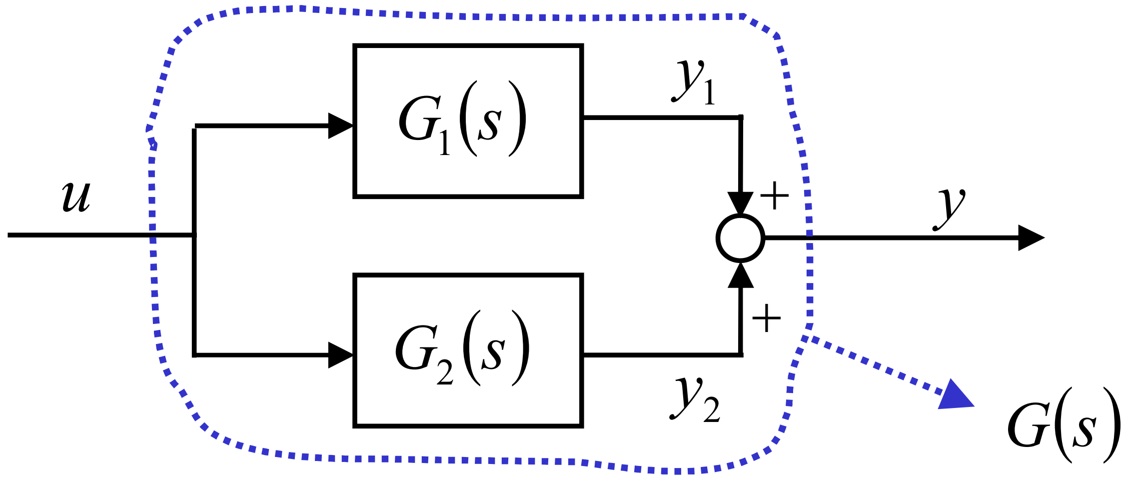
$$Y(s) = G_2(s)U_2(s) =$$

$$= G_2(s)Y_1(s) =$$

$$= G_2(s)G_1(s)U(s)$$

$$\frac{Y(s)}{U(s)} = G_2(s)G_1(s) = G(s)$$

Blocchi in parallelo



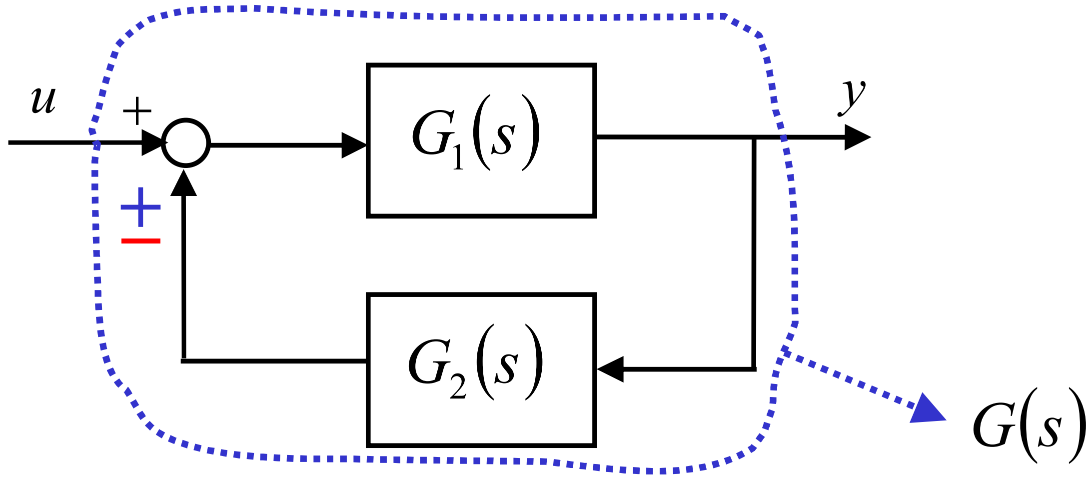
$$Y(s) = Y_1(s) + Y_2(s) =$$

$$= G_1(s)U(s) + G_2(s)U(s) =$$

$$= (G_1(s) + G_2(s))U(s)$$

$$\Rightarrow \quad \boxed{\frac{Y(s)}{U(s)} = G_1(s) + G_2(s) = G(s)}$$

Blocchi in retroazione



Retroazione

- negativa −
- positiva +

$$Y(s) = G_1(s)[U(s) \pm G_2(s)Y(s)]$$

$$[1 \mp G_1(s)G_2(s)]Y(s) = G_1(s)U(s)$$

$$\Rightarrow \quad \frac{Y(s)}{U(s)} = \frac{G_1(s)}{1 \mp G_1(s)G_2(s)} = G(s)$$

$$G(s) = \frac{\textbf{FdT “in andata”}}{1 \mp \textbf{FdT “d’anello”}}$$

Retroazione

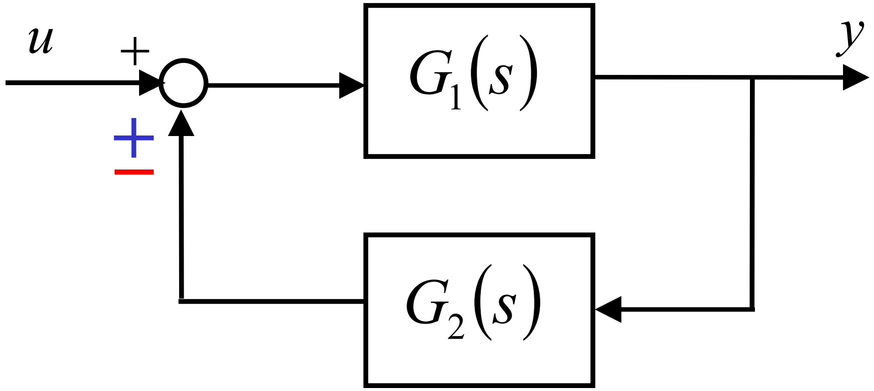
- negativa
- positiva

<u>FdT “in andata”</u> :

prodotto delle FdT da u a y in anello aperto

<u>FdT “d’anello”</u> :

prodotto delle FdT lungo l’anello.



Retroazione

- negativa $-$
- positiva $+$

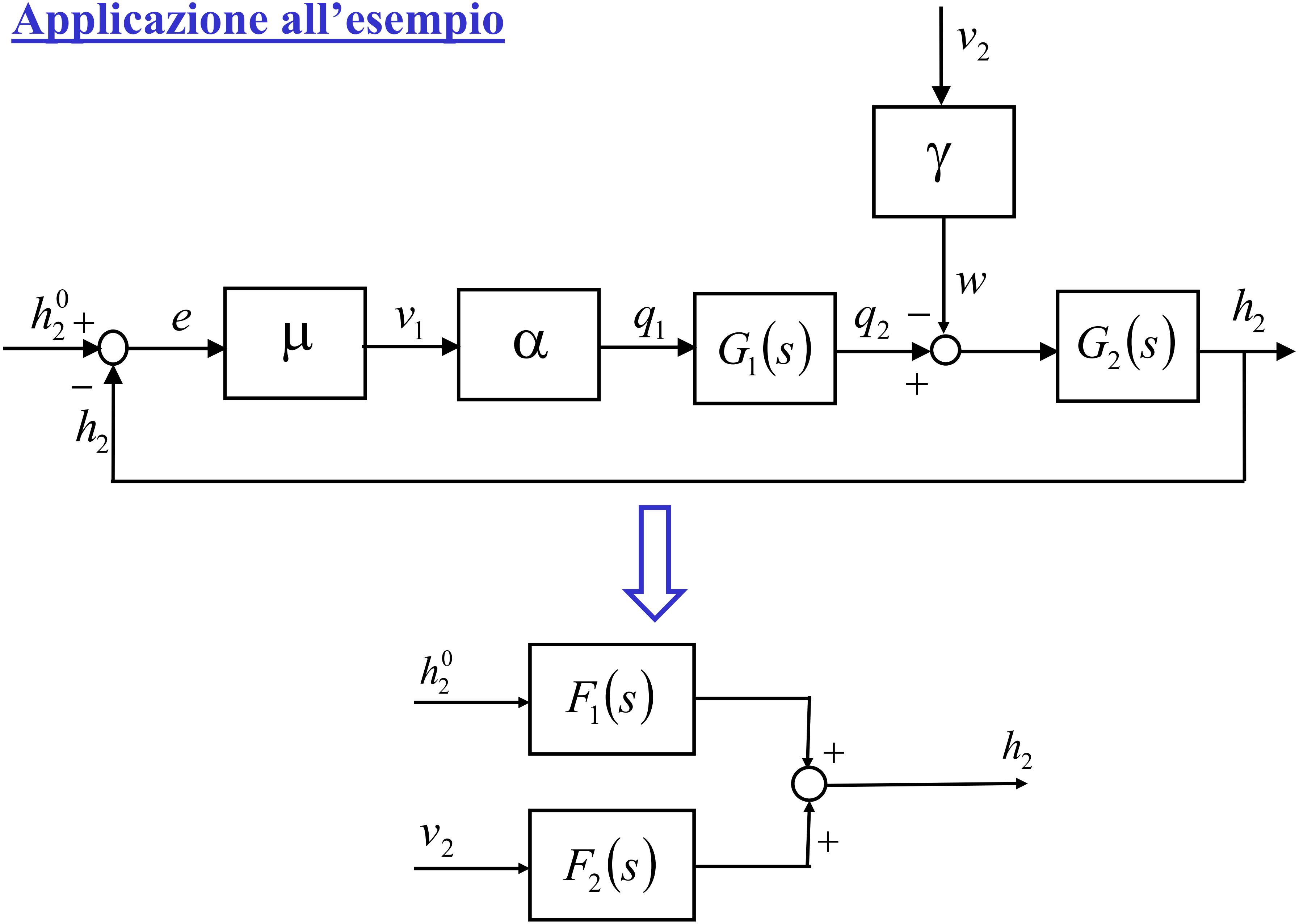
<u>**FdT "in andata"**</u> : $G_1(s)$

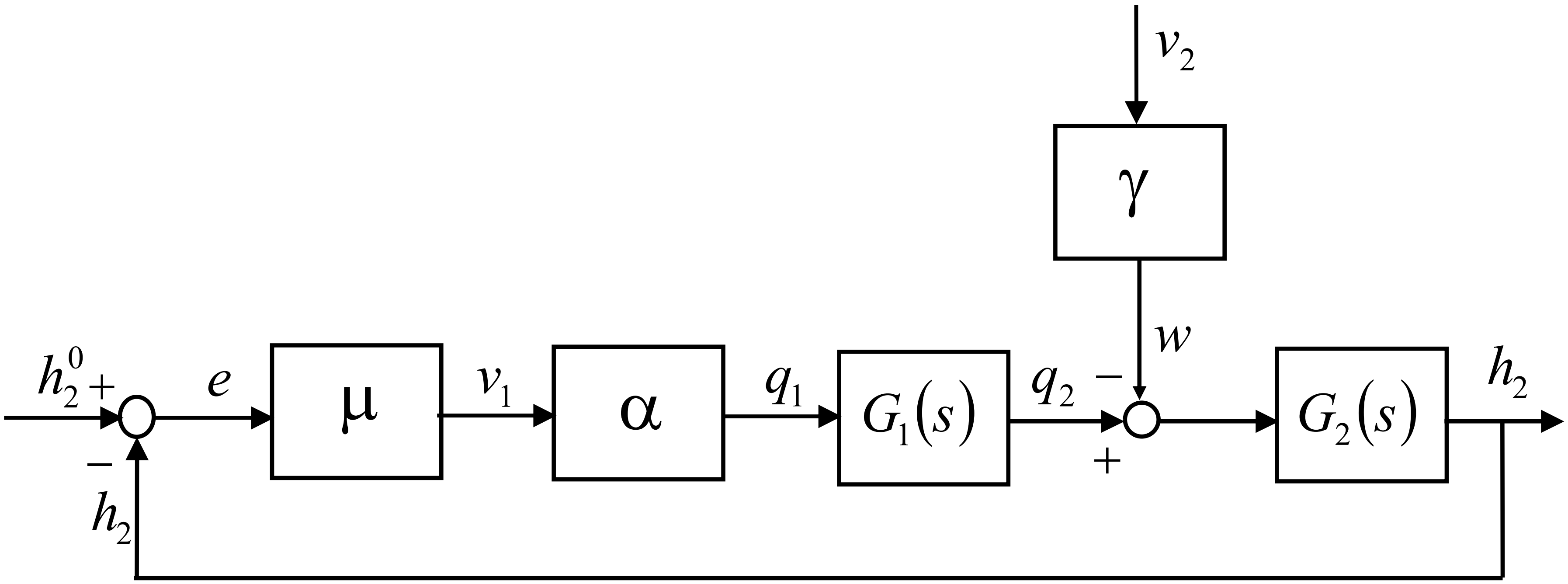
<u>**FdT "d'anello"**</u> : $G_1(s)G_2(s)$

$\Downarrow$

$$G(s)=\frac{G_1(s)}{1\mp G_1(s)G_2(s)}$$

Applicazione all'esempio





$$\frac{H_2(s)}{V_2(s)} = \frac{-\gamma G_2(s)}{1+\mu\alpha G_1(s)G_2(s)} = F_2(s)$$

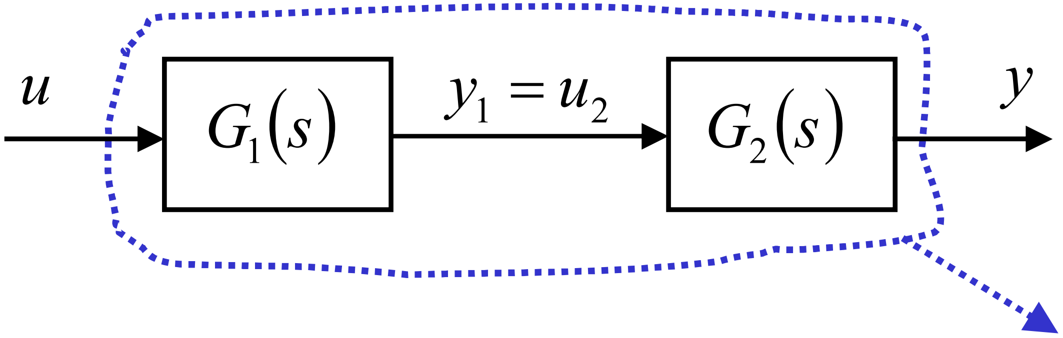
$$\frac{H_2(s)}{H_2^0(s)} = \frac{\mu\alpha G_1(s)G_2(s)}{1+\mu\alpha G_1(s)G_2(s)} = F_1(s)$$

4. Schemi a blocchi : stabilità

Blocchi

- in serie
- in parallelo
- in retroazione

Blocchi in serie



$$G(s) = G_1(s)G_2(s)$$

$$G_1(s) = \frac{N_1(s)}{D_1(s)} \qquad G_2(s) = \frac{N_2(s)}{D_2(s)}$$

$$G(s) = \frac{N_1(s)N_2(s)}{D_1(s)D_2(s)}$$

Senza cancellazioni

$$\left\{\begin{matrix}\text{poli di}\\ G(s)\end{matrix}\right\} = \left\{\begin{matrix}\text{poli di}\\ G_1(s)\end{matrix}\right\} \cup \left\{\begin{matrix}\text{poli di}\\ G_2(s)\end{matrix}\right\}$$

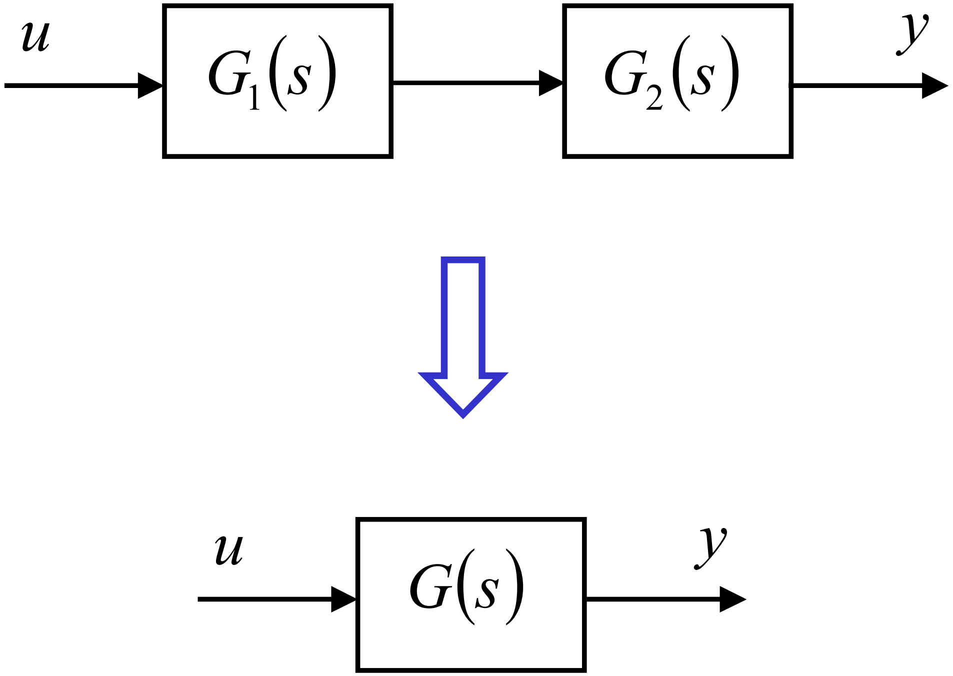
$G(s)$ As. stabile $\Longleftrightarrow$ $G_1(s), G_2(s)$ As. stabili

Con cancellazioni

- cancellazioni con $\mathrm{Re} < 0$ $\Rightarrow$ dinamica "nascosta" as. stabile
- cancellazioni con $\mathrm{Re} \geq 0$ $\Rightarrow$ dinamica "nascosta" non as. stabile

Il sistema non è as. stabile anche se $G(s)$ non lo mostra.

Esempio



$$G_1(s) = \frac{s+2}{s+1}$$

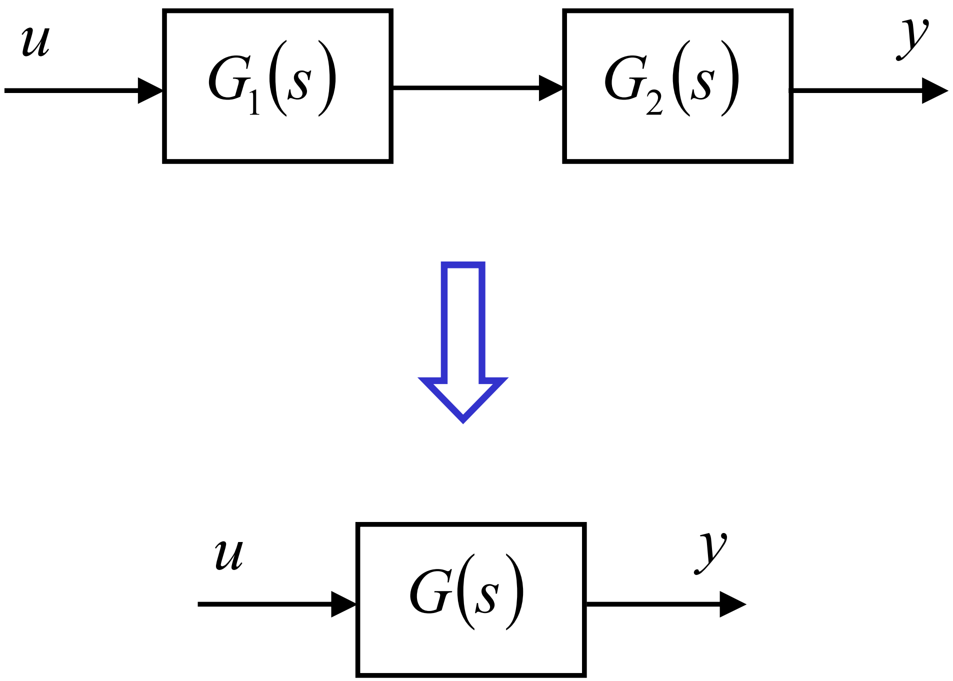
$$G_2(s) = \frac{1}{s+3}$$

as. stabili

$$G(s) = \frac{s+2}{(s+1)(s+3)}$$

as. stabile

Esempio

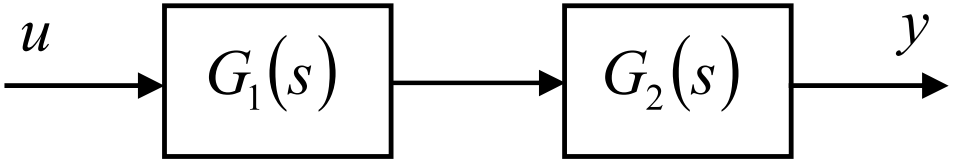


$$G_1(s) = \frac{s+2}{s-1}$$ instabile

$$G_2(s) = \frac{1}{s+3}$$ as. stabile

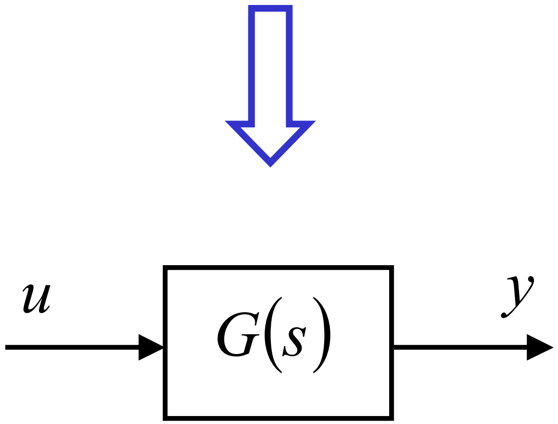
$$G(s) = \frac{s+2}{(s-1)(s+3)}$$ instabile

Esempio



$$G_1(s) = \frac{s+2}{s-1}$$ instabile

$$G_2(s) = \frac{s-1}{s+3}$$ as. stabile



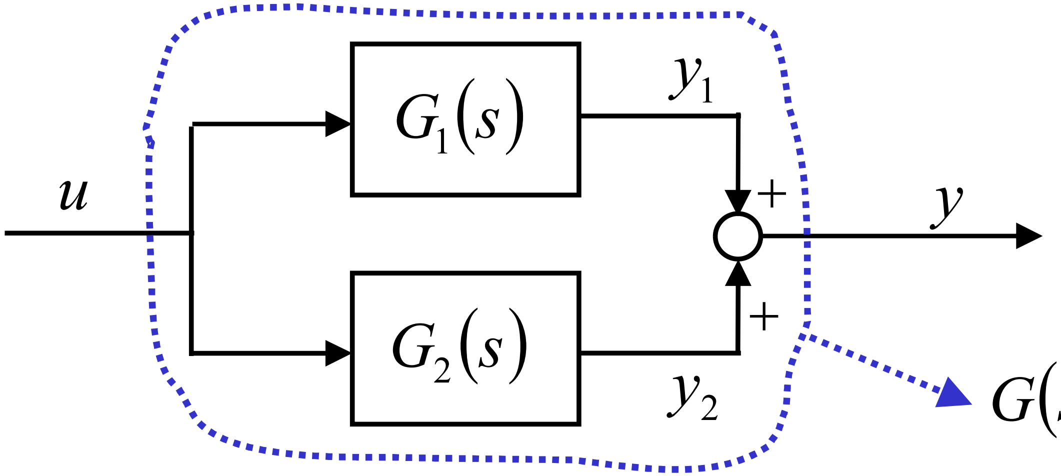
$$G(s) = \frac{(s+2)(s-1)}{(s-1)(s+3)}$$ instabile

C'è una cancellazione polo/zero instabile!

Il sistema è instabile!

(se eseguo la cancellazione, dalla funzione di trasferimento che risulta non si capisce, perchè la parte instabile del sistema viene "nascosta")

Blocchi in parallelo



$$G(s) = G_1(s) + G_2(s)$$

$$G_1(s) = \frac{N_1(s)}{D_1(s)} \qquad G_2(s) = \frac{N_2(s)}{D_2(s)}$$

$$G(s) = \frac{N_1(s)}{D_1(s)} + \frac{N_2(s)}{D_2(s)} = \frac{N_1(s)D_2(s) + N_2(s)D_1(s)}{D_1(s)D_2(s)}$$

Senza cancellazioni

$$\left\{\begin{matrix}\text{poli di}\\ G(s)\end{matrix}\right\} = \left\{\begin{matrix}\text{poli di}\\ G_1(s)\end{matrix}\right\} \cup \left\{\begin{matrix}\text{poli di}\\ G_2(s)\end{matrix}\right\}$$

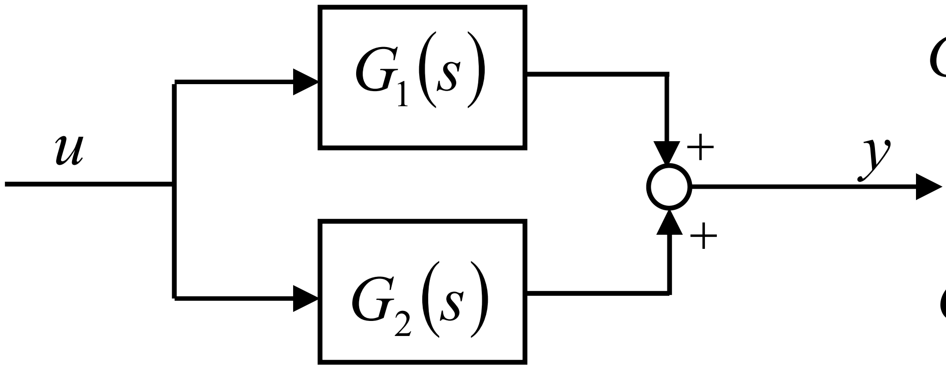
$$G(s)\ \begin{matrix}\text{As.}\\ \text{stabile}\end{matrix} \Longleftrightarrow G_1(s), G_2(s)\ \begin{matrix}\text{As.}\\ \text{stabili}\end{matrix}$$

Con cancellazioni

- cancellazioni con $\operatorname{Re} < 0$ $\Rightarrow$ dinamica "nascosta" as. stabile
- cancellazioni con $\operatorname{Re} \geq 0$ $\Rightarrow$ dinamica "nascosta" non as. stabile

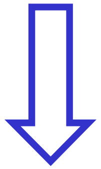
Il sistema non è as. stabile anche se $G(s)$ non lo mostra.

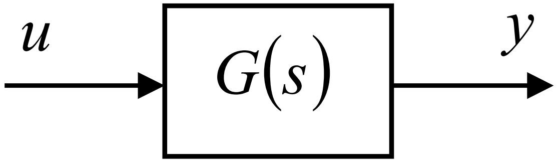
Esempio



$$G_1(s) = \frac{s}{s+1} \quad \text{as. stabile}$$

$$G_2(s) = \frac{1}{s-1} \quad \text{instabile}$$

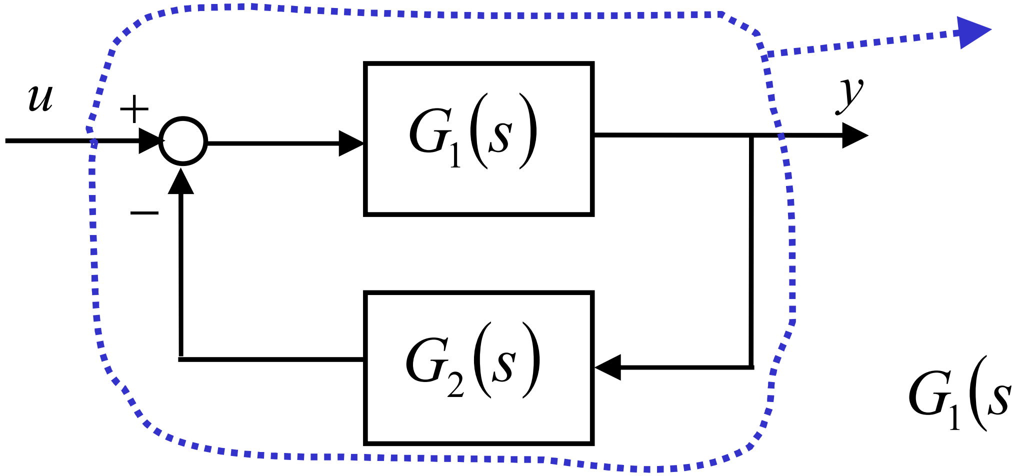




$$G(s) = \frac{s}{s+1} + \frac{1}{s-1} = \frac{s^2+1}{(s+1)(s-1)}$$

instabile

Blocchi in retroazione



$$G(s) = \frac{G_1(s)}{1 + G_1(s)G_2(s)}$$

$$G_1(s) = \frac{N_1(s)}{D_1(s)} \qquad G_2(s) = \frac{N_2(s)}{D_2(s)}$$

$$G(s) = \frac{N_1(s)/D_1(s)}{1 + \frac{N_1(s)N_2(s)}{D_1(s)D_2(s)}} = \frac{N_1(s)D_2(s)}{D_1(s)D_2(s) + N_1(s)N_2(s)}$$

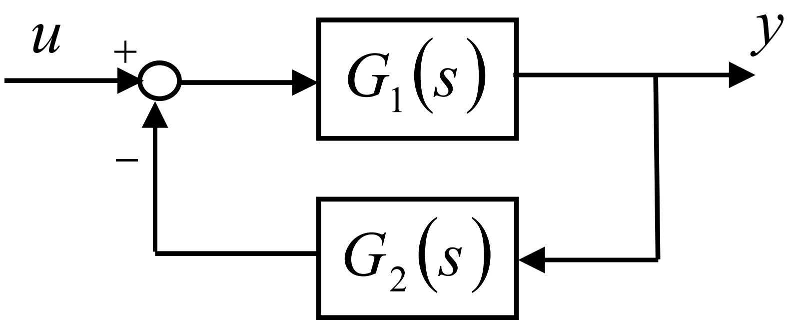
Senza cancellazioni

$$\left\{\begin{matrix}\text{poli di}\\ G(s)\end{matrix}\right\} = \left\{\begin{matrix}\text{radici di}\\ D_1(s)D_2(s)+N_1(s)N_2(s)=0\end{matrix}\right\}$$

Asintotica stabilità sistema complessivo $\not\Leftrightarrow$ Asintotica stabilità sottosistemi

$G(s)$ As. stabile $\not\Leftrightarrow$ $G_1(s), G_2(s)$ As. stabili

Esempio



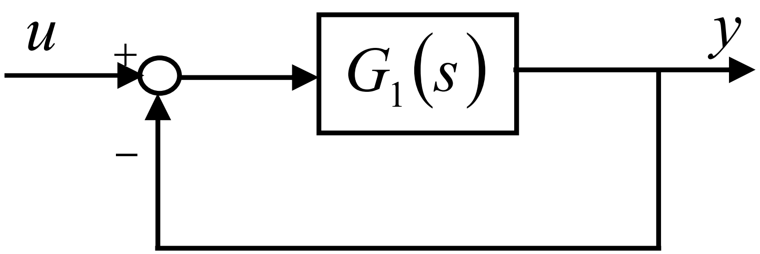
$$G_1(s) = \frac{9}{(s+1)^2}$$ **As. stabile**

$$G_2(s) = \frac{3}{(s+1)}$$ **As. stabile**

$$\frac{Y(s)}{U(s)} = G(s) = \frac{\frac{9}{(s+1)^2}}{1 + \frac{27}{(s+1)^3}} = \frac{9(s+1)}{(s+1)^3 + 27}$$

poli in $-4,\ \frac{1}{2} \pm j\frac{3\sqrt{3}}{2}$ **Instabile**

Esempio



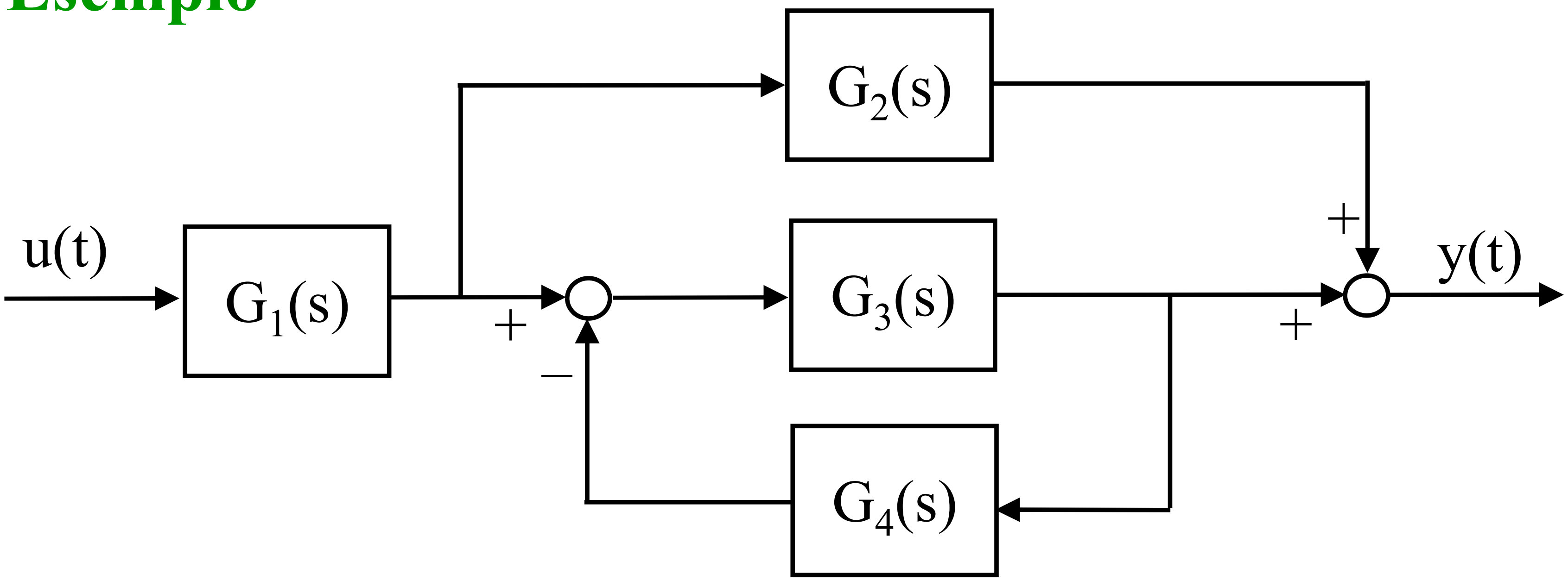
$$G_1(s) = \frac{10}{s-1}$$

Instabile

$$\frac{Y(s)}{U(s)} = G(s) = \frac{\frac{10}{s-1}}{1+\frac{10}{s-1}} = \frac{10}{s+9}$$

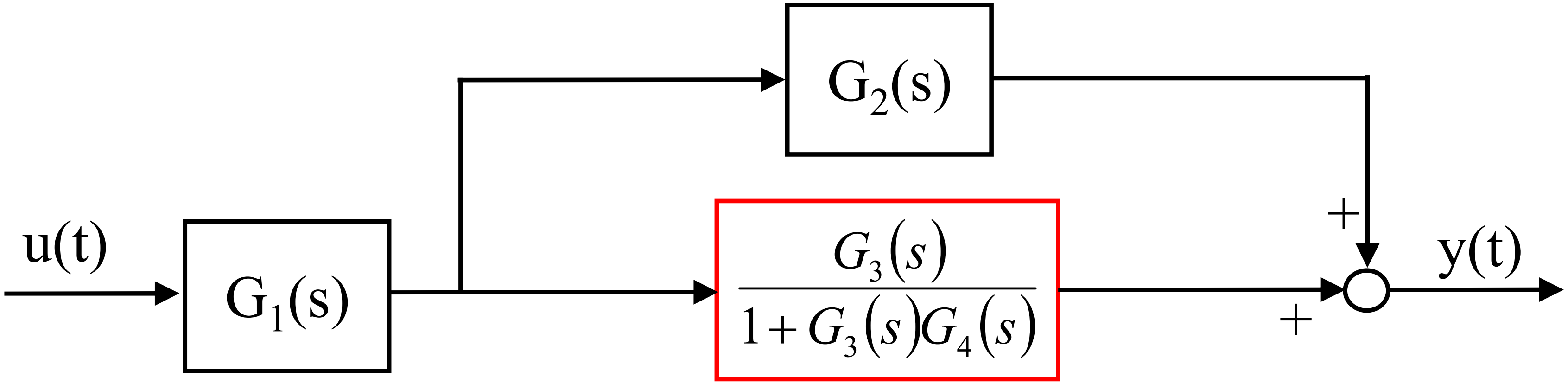
polo in -9 **As. stabile**

Esempio

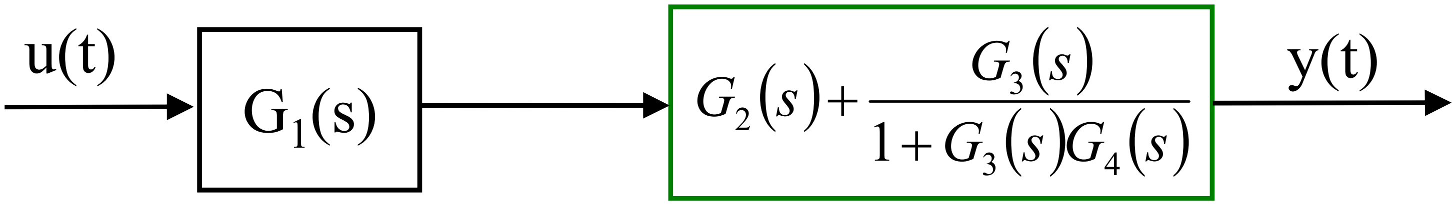


Calcolare la funzione di trasferimento da u(t) ad y(t)

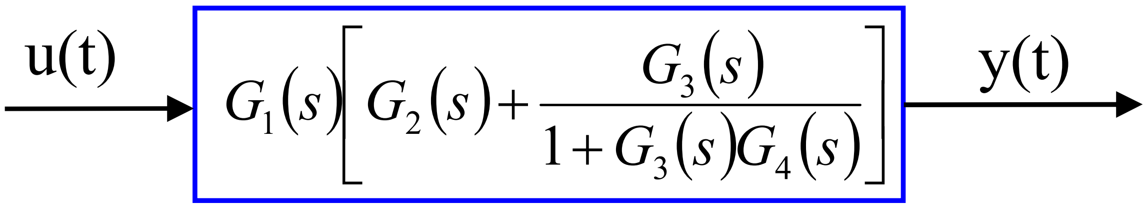
G_3 e G_4 sono in retroazione (negativa). Il risultato della loro connessione è in parallelo a G_2. Il risultato è in serie a G_1.



u(t)
$G_1(s)$
$G_2(s)$
$\frac{G_3(s)}{1+G_3(s)G_4(s)}$
+
+
y(t)

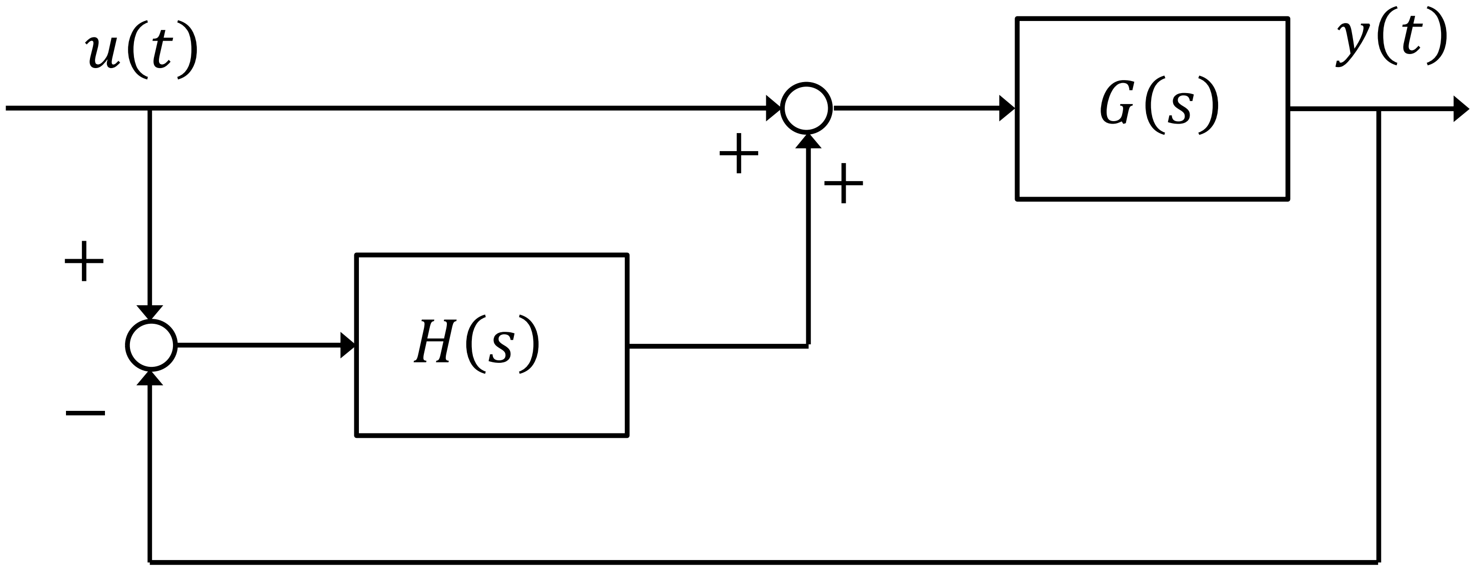


u(t)
$G_1(s)$
$G_2(s)+\frac{G_3(s)}{1+G_3(s)G_4(s)}$
y(t)



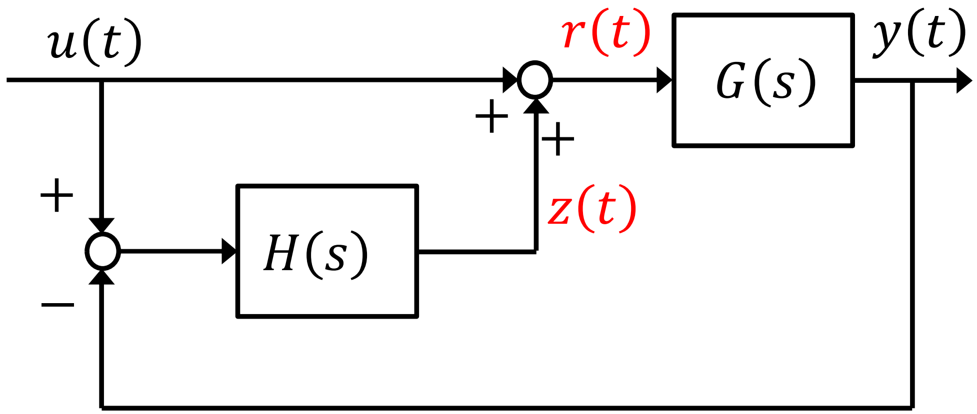
u(t)
$G_1(s)\left[G_2(s)+\frac{G_3(s)}{1+G_3(s)G_4(s)}\right]$
y(t)

Esempio



Calcolare la funzione di trasferimento da u(t) ad y(t).

Esistono altri modi per analizzare schemi a blocchi «complessi». Uno di questi prevede di attribuire dei nomi alle altre variabili in gioco e calcolare la funzione di trasferimento complessiva in modo diretto.



Chiamo $r(t)$ e $z(t)$ i due segnali senza nome.

$$Y(s) = G(s)R(s)$$

$$Y(s) = G(s)\big(U(s) + Z(s)\big) = G(s)U(s) + G(s)Z(s)$$

$$Y(s) = G(s)U(s) + G(s)H(s)\big(U(s) - Y(s)\big)$$

$$Y(s) = G(s)U(s) + G(s)H(s)U(s) - G(s)H(s)Y(s)$$

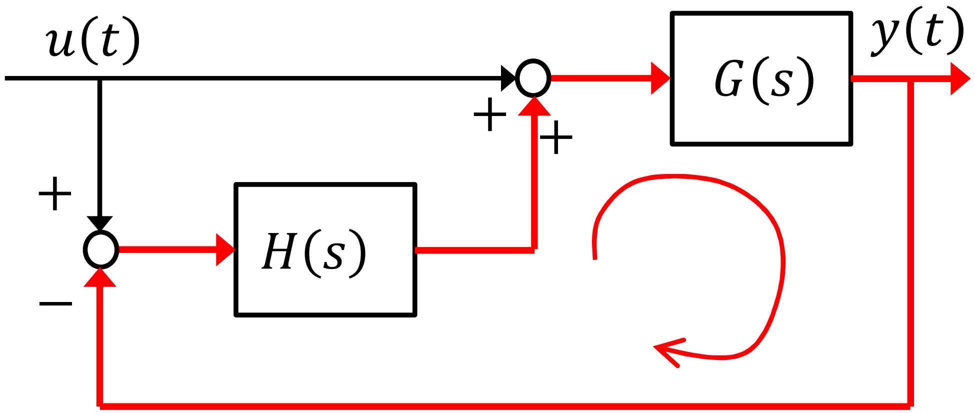
$$Y(s) = \big(G(s) + G(s)H(s)\big)U(s) - G(s)H(s)Y(s)$$

$$\big(1 + G(s)H(s)\big)Y(s) = \big(G(s) + G(s)H(s)\big)U(s)$$

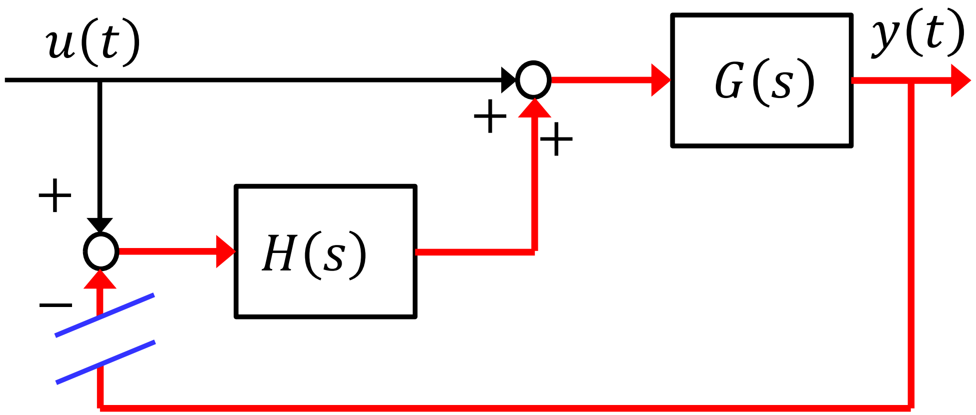
$$\frac{Y(s)}{U(s)} = \frac{G(s)\big(1 + H(s)\big)}{1 + G(s)H(s)}$$

E’ ovviamente possibile risolvere questo schema con il metodo «standard».

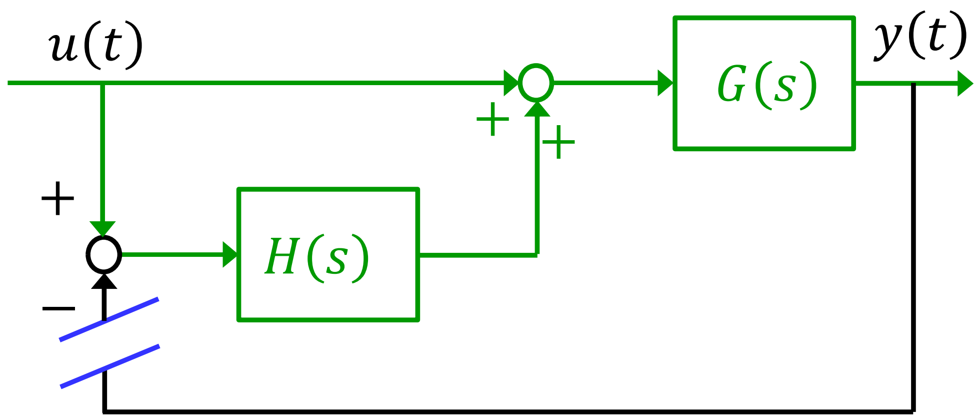
1) E’ un sistema retroazionato ed evidenzio l’anello.



2) Apro l’anello di retroazione.

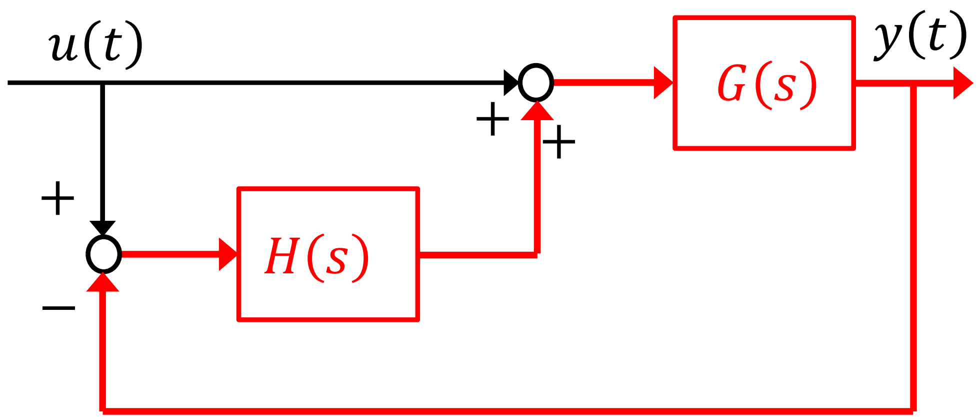


3) Calcolo la funzione di trasferimento d'andata (nota che ci sono due percorsi in parallelo).



$$A(s) = \big(1 + H(s)\big)G(s)$$

3) Calcolo la funzione di trasferimento d'anello.



$$L(s) = H(s)G(s)$$

4) Calcolo la funzione di trasferimento complessiva.

$$\frac{Y(s)}{U(s)} = \frac{FdT\ d'andata}{1 + FdT\ d'anello} = \frac{\big(1 + H(s)\big)G(s)}{1 + G(s)H(s)}$$

Il risultato è (ovviamente) identico al precedente.

5. Matlab

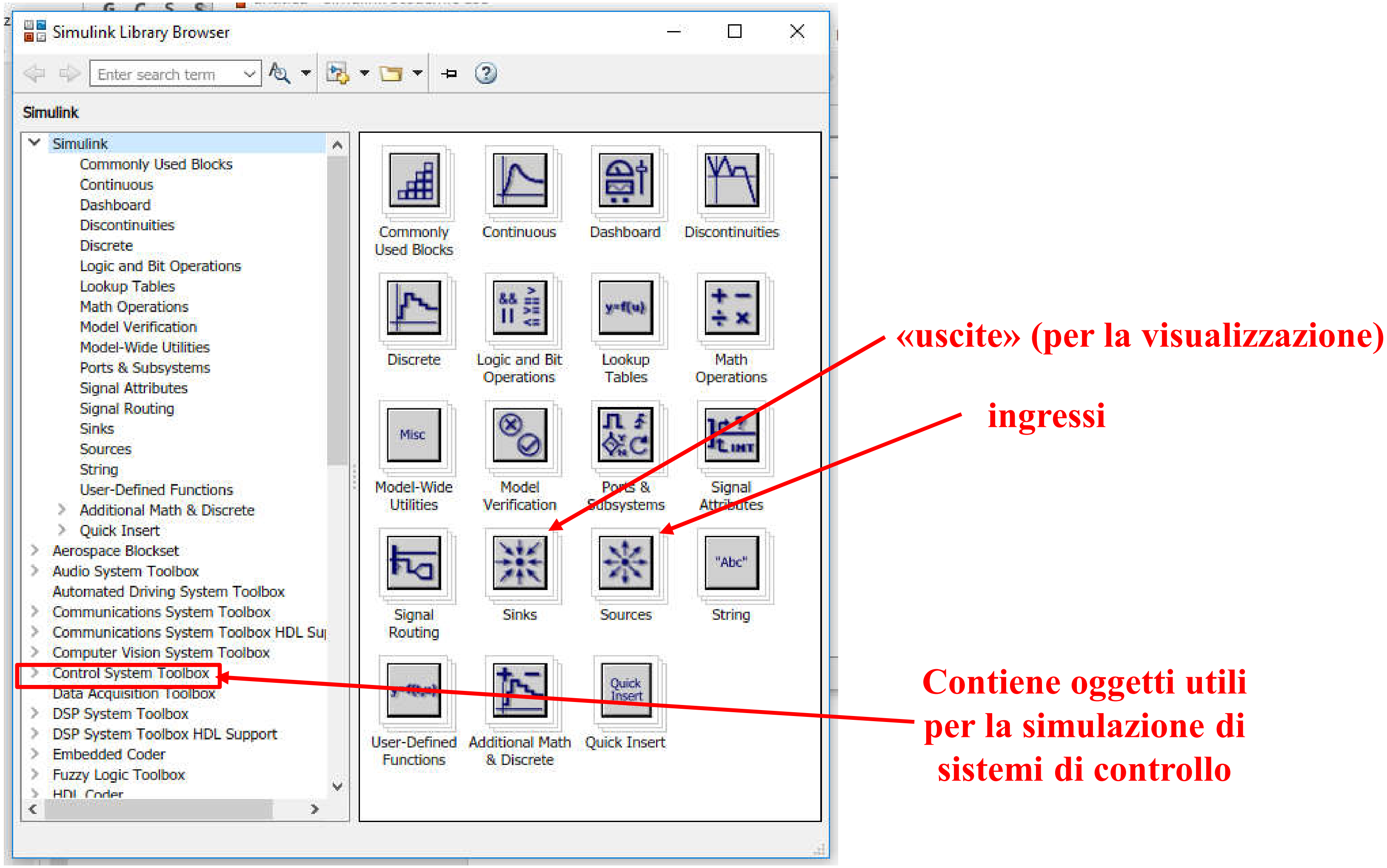
Matlab dispone di comandi per risolvere semplici schemi a blocchi in serie, parallelo o in retroazione.

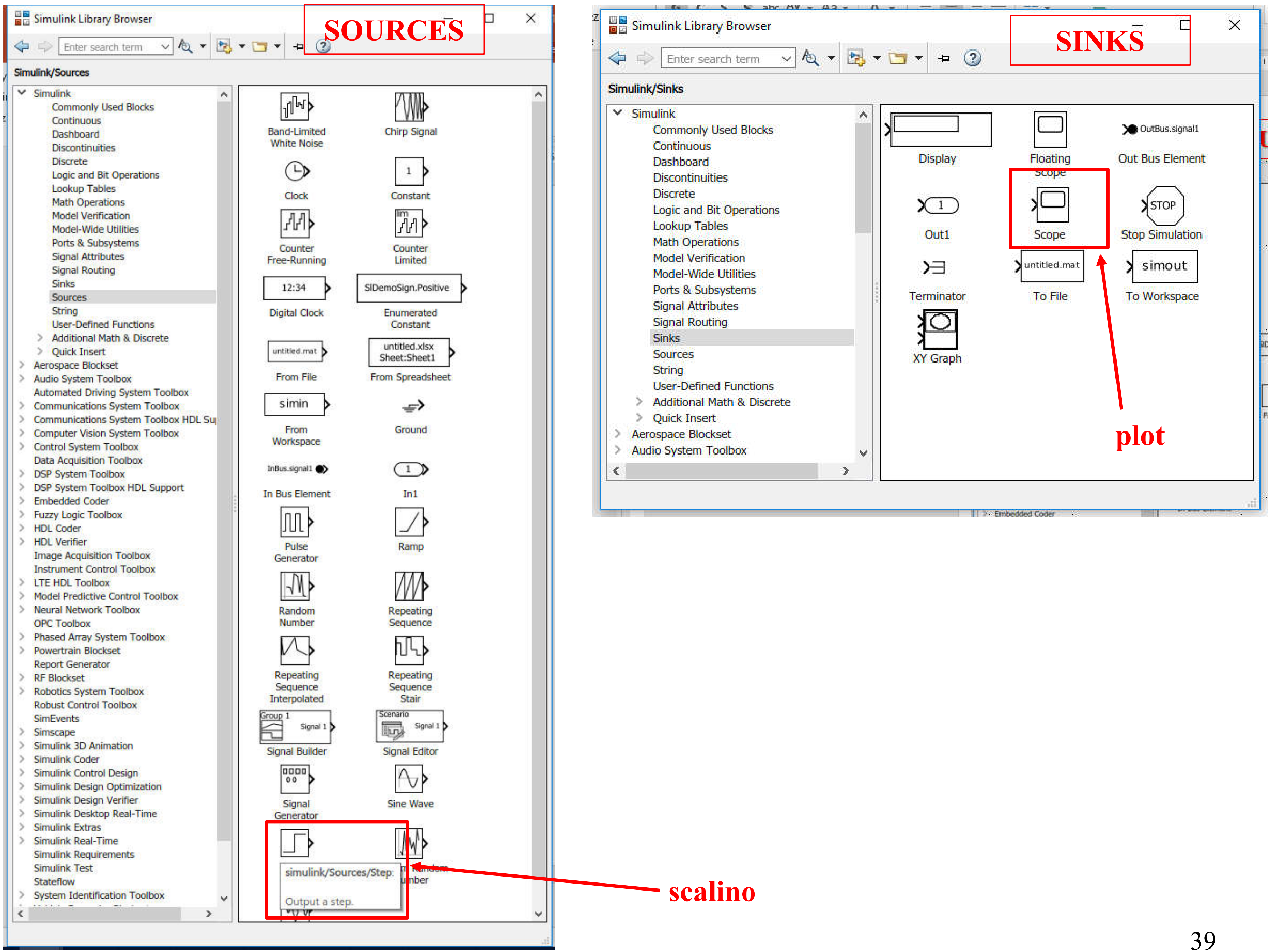
```
>> M = feedback(M1,M2);

>> M = series(M1,M2)

>> M = parallel(M1,M2)
```

6. Simulink

Simulink è un ambiente di calcolo matematico e simulazione basato sul paradigma degli schemi a blocchi.

Consente di creare modelli (`model`) utilizzando relazioni matematiche predefinite (anche non lineari!) e blockset/toolbox su diversi ambiti specifici.

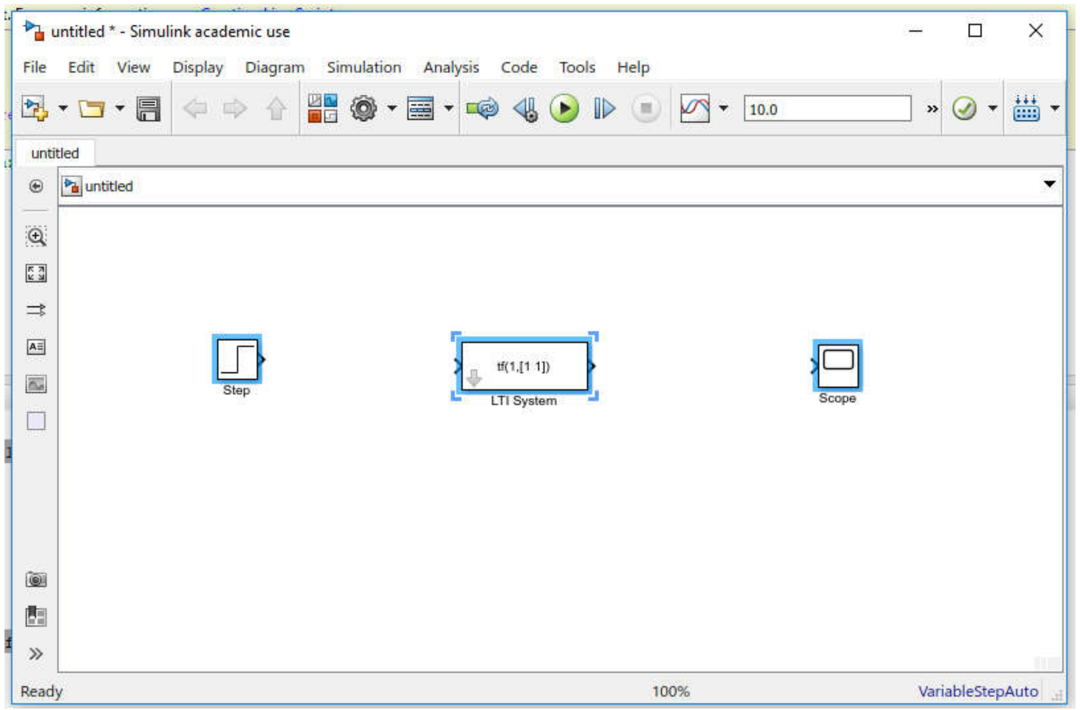



Simulink Library Browser
SOURCES
Simulink/Sources
Band-Limited White Noise
Chirp Signal
Clock
Constant
Counter Free-Running
Counter Limited
Digital Clock
Enumerated Constant
From File
From Spreadsheet
From Workspace
Ground
In Bus Element
In1
Pulse Generator
Ramp
Random Number
Repeating Sequence
Repeating Sequence Interpolated
Repeating Sequence Stair
Signal Builder
Signal Editor
Signal Generator
Sine Wave
simulink/Sources/Step
Output a step.
scalino
Simulink Library Browser
SINKS
Simulink/Sinks
Display
Floating Scope
Out Bus Element
Out1
Scope
Stop Simulation
Terminator
To File
To Workspace
XY Graph
plot

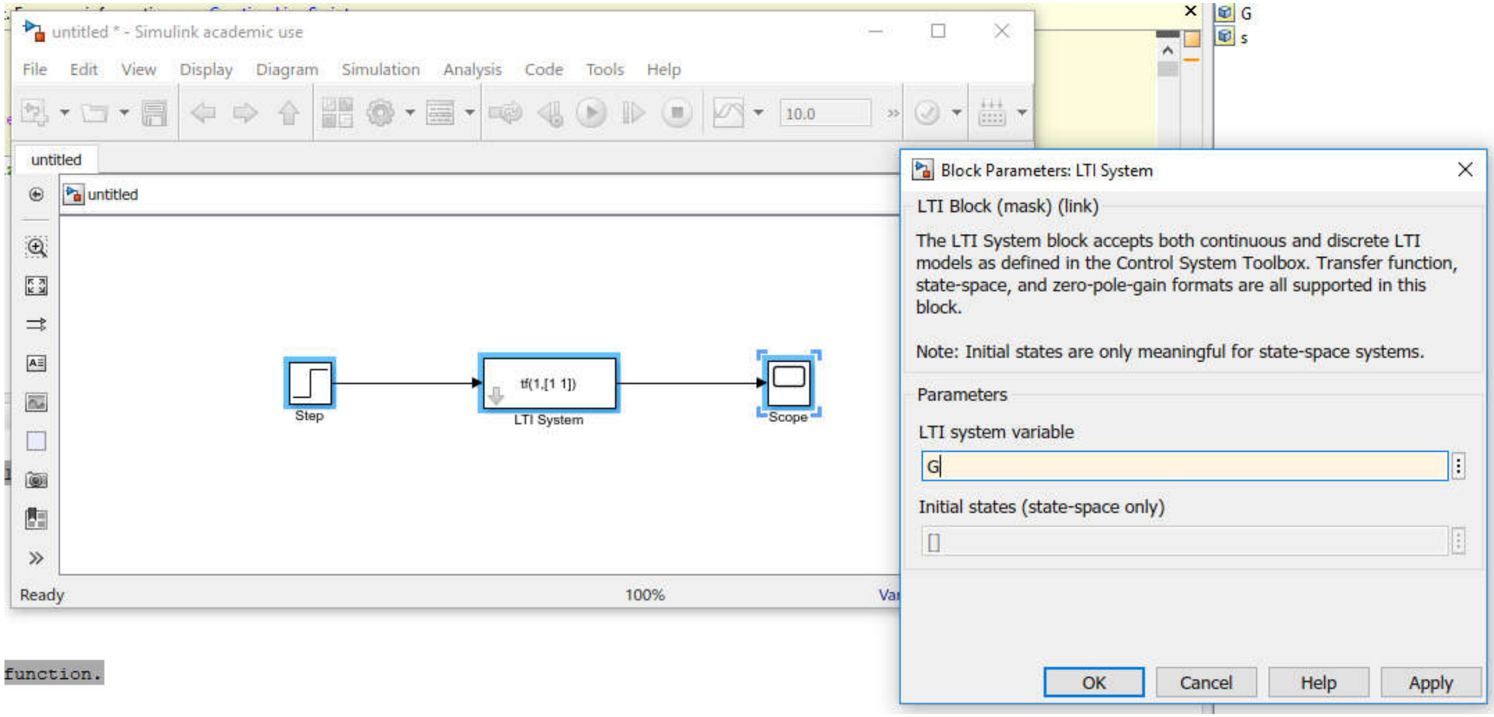
Definiamo nel Matlab Workspace una funzione di trasferimento

```
>> s=tf('s');
>> G=24*(s-0.1)/(s^2+2*s+100)

G =

    24 s - 2.4
  ---------------
  s^2 + 2 s + 100

Continuous-time transfer function.
```

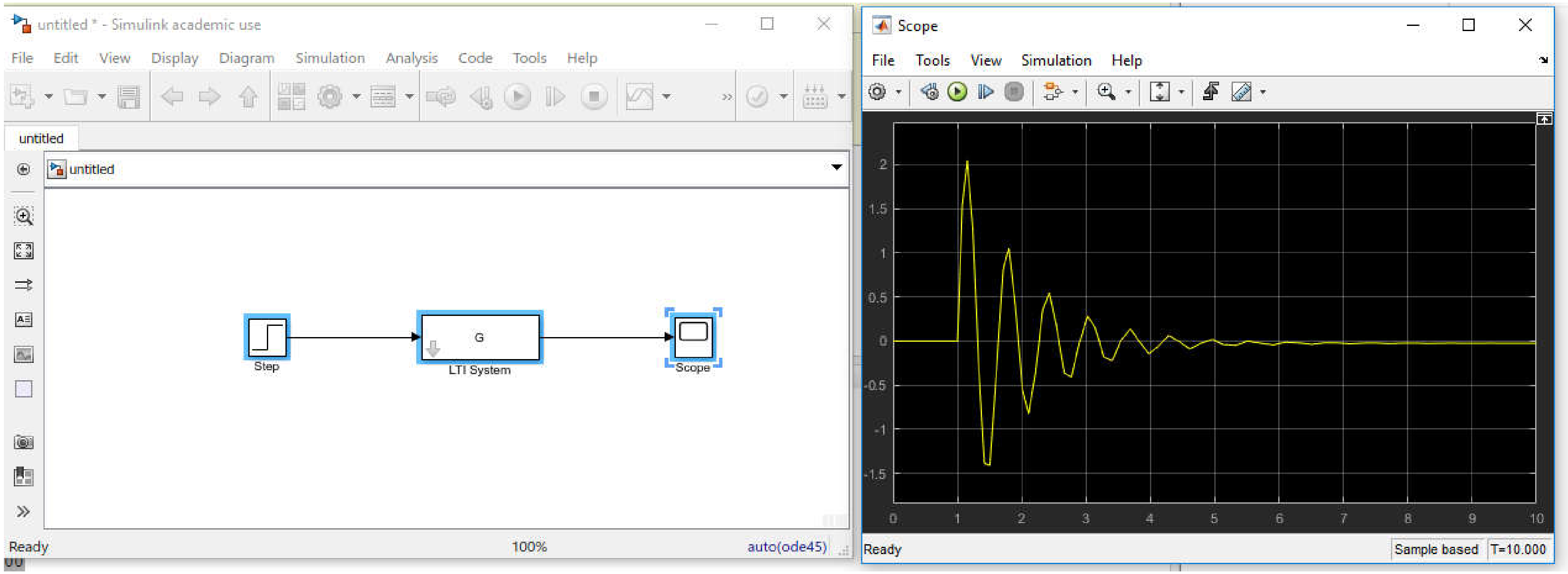
Creiamo un modello Simulink ed inseriamo una Source (per esempio Step), un Sink (per esempio Scope) ed un LTI model dal Control System Toolbox.



Li colleghiamo tra loro ed inseriamo nel LTI model la fdt G.



Apriamo la Scope e premiamo il pulsante Start.



E' possibile interconnettere (sotto-)sistemi semplici per ottenere sistemi complessi a piacere.