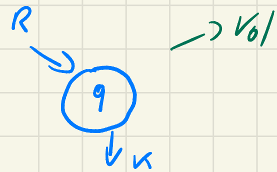
EX 1 | 1^{st} order elimination

$R = 200\,mg \qquad K = 0.105\ [h^{-1}]$

$\boxed{c(1) = 90\ mg/L}$



① Vol? CL?

$\dot{q}(t) = -K q(t) + R\delta(t) \qquad q(0) = 0$

$$\left\{\begin{array}{l} \dot{q}(t) = -K q(t) \\ q(t) = q(0)\, e^{-Kt} = R e^{-Kt} \end{array}\right. \qquad q(0)^{+} = R$$

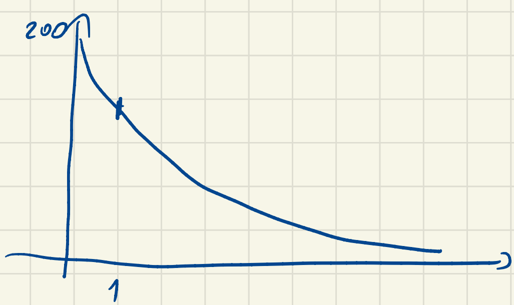
$\dot{x}(t) = A x(t) \rightarrow x(t) = x(0) e^{At}$ FREE MOVEMENT

$A = -K \qquad K > 0 \Rightarrow -K < 0 \Rightarrow$ A.S.

As $t \rightarrow \infty \quad q(t) \rightarrow 0$

$c(1) = \dfrac{q(1)}{Vol} \Rightarrow Vol = \dfrac{q(1)}{c(1)}$

$\boxed{q(t) = R e^{-Kt}} \Rightarrow q(1) = R e^{-K} = 200\, e^{-0.105} \approx 180\,mg$



$Vol = \dfrac{q(1)}{c(1)} = \dfrac{180}{90} = 2\ [L]$

$CL = Vol \cdot K = 0.210\ [L/h]$

(2) $c(2)$?

$$q(t) = R e^{-Kt}$$

$$c(2) = \frac{q(2)}{V_d} = \frac{R}{V} e^{-Kt}$$

$$c(2) = \frac{200}{2} e^{-0.210} = 100\, e^{-0.210} = 100 \cdot 0.81 = 81\,[mg/L]$$

(3) $\hat{t}$ s.t. $c(\hat{t}) = 50\,[mg/L]$

$$c(\hat{t}) = \frac{q(\hat{t})}{V} = \frac{R}{V} e^{-K\hat{t}} \qquad c(\hat{t}) = 50\ [mg/L]$$

$$\frac{200}{2} e^{-0.105\hat{t}} = 50 \Rightarrow 100 \cdot e^{-0.105\hat{t}} = 50$$

$$e^{-0.105\hat{t}} = 0.5 \Rightarrow -0.105\hat{t} = \ln(0.5)$$

$$\hat{t} = -\frac{\ln(0.5)}{0.105} = \frac{0.693}{0.105} \approx 6.6\,[h]$$

half-life

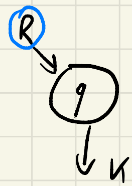
$$t_{1/2} = \frac{\ln(2)}{0.105} = \frac{0.693}{0.105} = 6.6\,[h]$$

$$t_{1/2} = t \text{ s.t. } \bar{q} = \frac{R}{2}$$

$$q(t_{1/2}) = \cancel{R} e^{-Kt_{1/2}} = \frac{\cancel{R}}{2} \Rightarrow \cancel{R}\ e^{-Kt_{1/2}} = \frac{1}{2}$$

EX 2) $R = 400\ mg$ BOLUS

$C(0) = 200\ mg/L$ $K = 0.1\ [h^{-1}]$



(1) $Vol\ ?$ $CL\ ?$

$$\dot{q}(t) = -Kq(t) + R\,\delta(t)$$

$$\approx \boxed{\dot{q}(t) = -Kq(t)} \qquad q(0) = R \qquad C(0) = 200\ mg/L$$

$$Vol = \frac{q(0)}{C(0)} = \frac{400}{200} = 2\ [L]$$

$$CL = K \cdot Vol = 0.1 \times 2 = 0.2\ [L/h]$$

(2) $C(2)$ $t = \underline{2h}$ $t = 0$

$$\boxed{q(t) = q(0)\,e^{-Kt}} \qquad C(t) = \frac{q(0)}{Vol}\,e^{-Kt} = C(0)\,e^{-Kt}$$

$$C(2) = 200 \times e^{-0.2} = 200 \times 0.8187 \approx 163.75\ [mg/L]$$

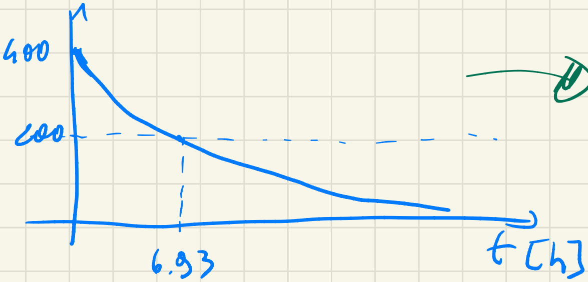
(3) $t_{1/2}\ ?$

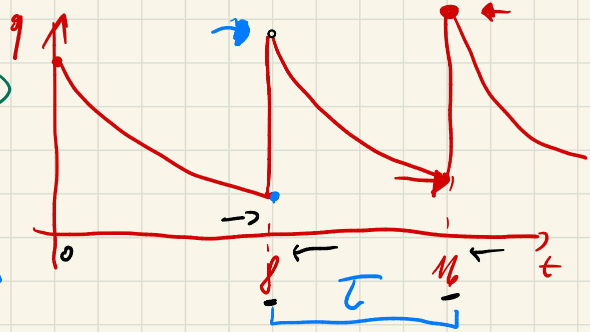
$$\boxed{t_{1/2} = \frac{\ln(2)}{K}} = \frac{0.693}{0.1} = 6{,}93\ [h]$$

$$q(t_{1/2}) = q(0)\,e^{-Kt_{1/2}} \qquad q(t_{1/2}) = \frac{q(0)}{2}$$

$$\frac{\cancel{q(0)}}{2} = \cancel{q(0)}\,e^{-Kt_{1/2}} \qquad \frac{1}{2} = e^{-Kt_{1/2}}$$

$$2 = e^{Kt_{1/2}} \Rightarrow \ln(2) = K\,t_{1/2} \Rightarrow t_{1/2} = \frac{\ln(2)}{K}$$





Multiple injection every 8h $\Rightarrow \tau = 8\ h$

(4) $c(8^+)$ $\qquad c(16^+)$

$$c(8^+) = c(8^-) + c(0) \qquad q(8^+) = q(8^-) + R$$

$$q(8^-) = q(0)\, e^{-K\cdot 8}$$

$$= 400\, e^{-0.8} = 179.73\ [mg]$$

$$q(8^+) = q(8^-) + R = 179.73 + 400 = 579.73\ mg$$

$$c(8^+) = \frac{q(8^+)}{V_d} = 289.87\ [mg/L]$$

$$c(16^+) = c(16^-) + c(0) \qquad q(16^+) = q(16^-) + R$$

$$q(j\cdot\tau^-) = q((j-1)\tau^+)\, e^{-K\tau} \qquad q(16^-) = q(8^+)\, e^{-K\times 8}$$

$$q(16^-) = 579.73\, e^{-0.8} = 260.43\ mg.$$

$$q(16^+) = q(16^-) + R = 660.43\ mg$$

$$c(16^+) = \frac{q(16^+)}{V_d} = 330{,}245\ mg/L$$

⑤ Time necessary to reach the steady-state

$\longrightarrow t_{ss} = 5 \cdot \underline{t_{1/2}}$

$\overset{!}{=} 5 \cdot 6.93 \approx 35\ h$

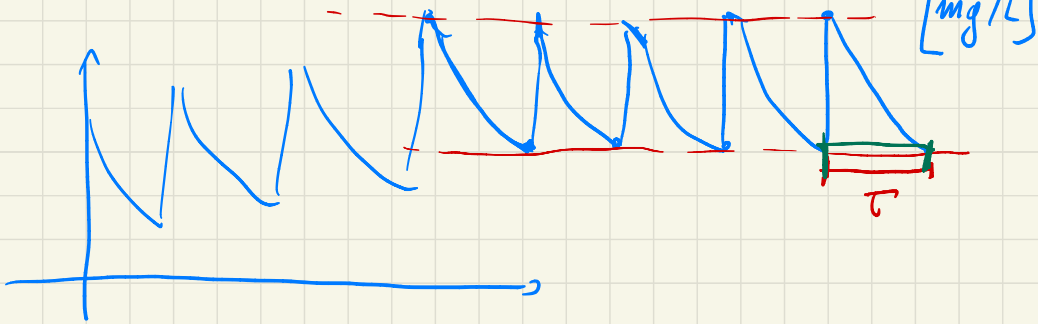
⑥ C_{ss} ?

$$C_{ss} = \frac{R \cdot F}{\underbrace{Vol \cdot K_i}_{CL} \cdot \tau} = \frac{RF}{CL \cdot \tau}$$

F = 1

$$\bar{C}_{ss} = \frac{R}{CL \cdot \tau} = \frac{400}{0.2 \times 8} = \frac{400}{1.6} = 250$$

[mg/L]



⑦ $\bar{C}_{ss}$ if $\tau = 6h$? $\bar{C}_{ss}$ if $\tau = 12\ h$?

$$\bar{C}_{ss}^{6} = \frac{R}{CL \cdot 6} = \boxed{333{,}33}$$

[mg/L]

$$\bar{C}_{ss}^{12} = \frac{R}{CL \cdot 12} = 166{,}67$$

[mg/L]

$\bar{C}_{ss}^{8} = 250$ [mg/L]

⑧ $\bar{C}_{ss}^{8} = 250$ [mg/L]

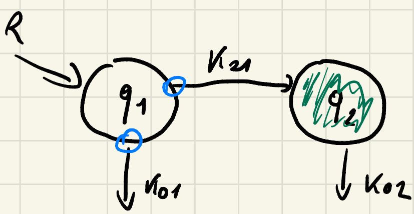
$\bar{C}_{ss}^{N} = 1.2 \times 250 = 300$ [mg/L]

$C_{ss}^{N} = \frac{R}{CL \cdot \tau} \Rightarrow \tau^{N} = \frac{R}{CL \cdot C_{ss}^{N}}$

$= \frac{400}{0.2 \times 300} = 6,67$ [h]

$\tau = 7h$ $\tau = 6\ h$

Ex 3



R = 500 mg
(oral adm)

F = 0.85

$K_{02} = 0.1$ [h^{-1}]

2 hours after, the remaining dose in the absorp. compart.
250 mg.

① Bioavailable dose? $D_B = F \times R = 425$ mg

② $K_a = ?$ $K_a = K_{01} + K_{21}$

$F = \frac{K_{21}}{K_a}$

$$\begin{cases} \dot{q}_1(t) = -\underbrace{(K_{01}+K_{21})}_{Ka}\, q_1(t) = -Ka\, q_1(t) \\ \dot{q}_2(t) = K_{21}\, q_1(t) - K_{02}\, q_2(t) \end{cases}$$

$q_1(0) = R$

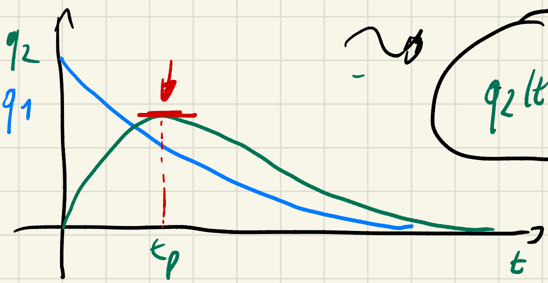
$q_2(0) = 0$

$$\boxed{q_1(t) = q_1(0)\, e^{-Ka\, t}}$$

$q_1(0) = 500$ mg $\qquad q_1(2) = 250$ mg

$\hookrightarrow t_{1/2} = 2$ h

$$t_{1/2} = \frac{\ln(2)}{Ka} \Rightarrow Ka = \frac{\ln(2)}{t_{1/2}} = \frac{0.693}{2} \approx 0{,}3465 \; [h^{-1}]$$



$\sim \emptyset$

$$\boxed{q_2(t) = \frac{RFKa}{K_{02}-Ka}\left(e^{-Ka\, t} - e^{-K_{02}\, t}\right)}$$

$$\boxed{t_p = \frac{\ln(K_{02}) - \ln(Ka)}{K_{02} - Ka}}$$

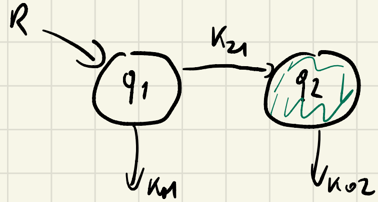
③ t_{peak}? $\quad c_{peak}$? $\quad (Vol = 3L)$

$$t_p = \frac{\ln(0{,}1) - \ln(0{,}3465)}{0.1 - 0.3465} \approx \frac{-1{,}24}{-0{,}2465} \approx 5 \; [h]$$

$$c_p = \frac{q_p}{Vol} \Rightarrow q_p = q_2(t_p) = \frac{RFKa}{K_{02}-Ka}\left(e^{-Ka \times t_p} - e^{-K_{02} \times t_p}\right)$$

$q_p = 256.56$ mg $\qquad c_p = \frac{q_p}{3} = 85.52 \; [mg/L]$

ex 4)



$R = 500$ mg

$F = 0.8$

$K_{02} = 0.2$ $[h^{-1}]$

3 h after intake, the remaining dose in ab. comp. 250 mg

① $D_B = ?$

$$D_B = 0.8 \cdot R = 400 \text{ mg}$$

② $K_e = ?$

$$F = \frac{K_{21}}{K_e} \qquad K_e = K_{01} + K_{21}$$

$$\begin{cases} \dot{q}_1(t) = -\underbrace{(K_{01} + K_{21})}_{K_e} q_1(t) = -K_e q_1(t) \\ \dot{q}_2(t) = K_{21} q_1(t) - K_{02} q_2(t) \end{cases}$$

$q_1(0) = 500$

$q_2(0) = 0$

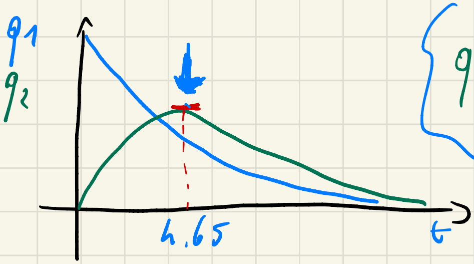
$$A = \begin{bmatrix} -K_{01} - K_{21} & 0 \\ K_{21} & -K_{02} \end{bmatrix}$$

$q_1(3) = 250$ mg $\Rightarrow t_{1/2} = 3h$

$$t_{1/2} = \frac{\ln(2)}{K_e} \Rightarrow K_e = \frac{\ln(2)}{t_{1/2}} = \frac{0.693}{3} = 0.231 \; [h^{-1}]$$

$$K_{21} = F \cdot K_e = 0.1848 \; [h^{-1}]$$

③ t_p ? c_p ? $V_d = 2\,L$



$$\boxed{q_2(t) = \frac{RFK_a}{K_{02} - K_a}\left(e^{-K_a t} - e^{-K_{02} t}\right)}$$

$$\boxed{t_p \Rightarrow \frac{d\, q_2(t)}{dt} = 0}$$

$$\dot{q}_2(t) = 0 \Rightarrow \quad K_{21}\,\bar{q}_1 - K_{02}\,\bar{q}_2 = 0$$

$$\underline{K_{21}\,\bar{q}_1} = \underline{K_{02}\,\bar{q}_2}$$

$$t_p = \frac{\ln(K_{02}) - \ln(K_a)}{K_{02} - K_a} = \frac{\ln(0.2) - \ln(0.231)}{0.2 - 0.231} = \underline{4.65\,[h]}$$

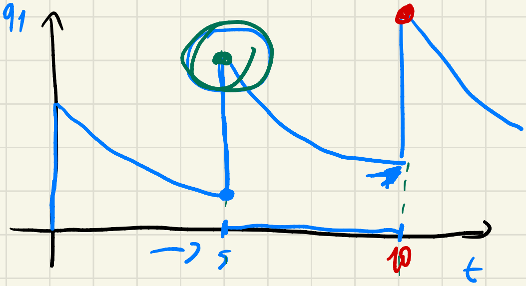
$$q_p = q_2(t_p) = \frac{RFK_a}{K_{02} - K_a}\left(e^{-K_a \times t_p} - e^{-K_{02} \times t_p}\right)$$

$$= 157.97\,[mg] \approx 158\,[mg]$$

$$c_p = \frac{q_p}{V_d} = \frac{158}{2} = 79\,[mg/L]$$

④ $\tau = 5h$ (Multiple dose)

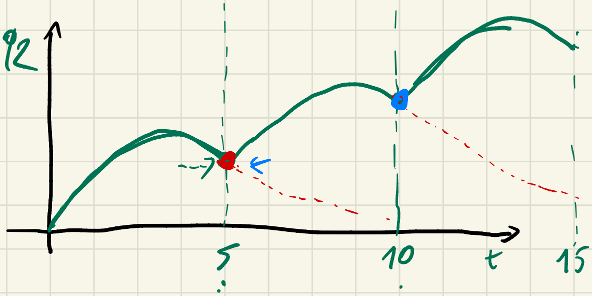
$c(5^+)$? $c(10^+)$? $c(15^+)$?



The same as before

$q_1(5^+) = q_1(5^-) + R$

$q_1(5^-) = q_1(0)\, e^{-k_a \tau}$



$t = 5$ $\quad q_1(5^-) \neq q_1(5^+) \Rightarrow q_1(5^+) = q_1(5^-) + R$

$q_2(5^-) = q_2(5^+)$

$q_1(5^-) = q_1(0)\, e^{-k_a \times 5} = 500\, e^{-0.231 \times 5} = 157.53$ mg

$q_1(5^+) = q_1(5^-) + 500 = 657.53$ mg

$q_2(5^-) = \dfrac{RFk_a}{k_{02} - k_a}\left(e^{-k_a \times \tau} - e^{-k_0 \times \tau}\right) = 157.44$ mg

$C_2(5^-) = \dfrac{157.44}{2} = \boxed{78.72\ [mg/L]}$

$= c_2(5^+)$

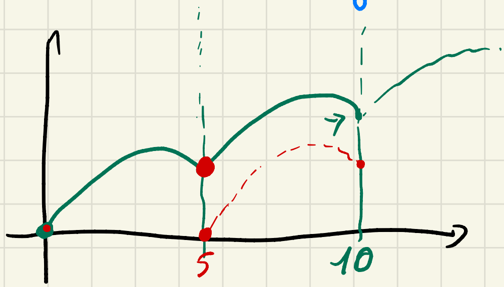
$t = 10h \Rightarrow$ Third dose

$$q_1(10^-) = q_1(5^+)\, e^{-K_a \times \tau}$$

$$= \underline{657.53}\, e^{-0.231 \times 5} = 207.16 \text{ mg}$$

$$q_1(10^+) = q_1(10^-) + R = 707.16 \text{ mg}$$

$q_2(10^-)$?



$$q_2(10^-) = \underbrace{\frac{q_1(5^+) F K_a}{K_{02} - K_a}\left(e^{-K_a \tau} - e^{-K_{02}\tau}\right)}_{\text{T.E. abs. + dist of the new doses}} + \underbrace{q_2(5)\, e^{-K_{02}\tau}}_{\text{Elimination of what was remaining from the previous dos}}$$

$$= \frac{657.53\, F K_a}{K_{02} - K_a}\left(e^{-K_a \times 5} - e^{-K_{02} \times 5}\right) + 157.44\, e^{-K_{02} \times 5}$$

$$= 264.97 \text{ mg}$$

$$\stackrel{!}{=} q_2(10^+) \qquad C_2(10^+) = \frac{q_2(10^+)}{Vol} = 132.485 \text{ [mg/L]}$$

$C_2(0) = 0 \qquad C_2(5^+) = 78 \text{ [mg/L]} \qquad C_2(10^+) = 132.485 \text{ [mg/L]}$

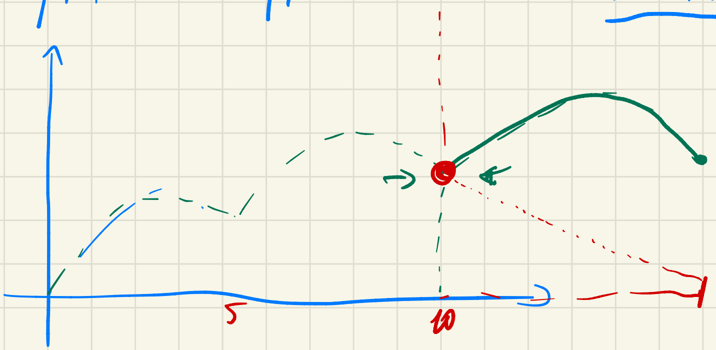
4th dose is at time $t = 15$

$$q_1(15^+) = \underbrace{q_1(15^-)} + R$$

$$q_1(15^-) = \underline{q_1(10^+)}\, e^{-K_a \cdot \tau}$$

$$= 707.16\, e^{-0.231 \times 5} = 222.736 \text{ mg}$$

$$q_1(15^+) = q_1(15^-) + 500 = \underline{722.736} \text{ mg}$$



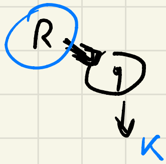
$$q_2(15^-) = \frac{q_1(10^+)\, F K_a}{K_{02} - K_a}\left(e^{-K_a \times \tau} - e^{-K_{02} \times \tau}\right) + q_2(10^+)\, e^{-K_{02} \times \tau}$$

$$= \frac{707.16\, F K_a}{K_{02} - K_a}\left(e^{-K_a 5} - e^{-K_{02} 5}\right) + 264.97\, e^{-K_{02} 5}$$

$$= 320.15 \text{ mg}$$

$$c_2(15^-) = c_2(15^+) = 160.075 \; [\text{mg}/L]$$

Ex 5



$R = 36$ mg/h

$\bar{C} = 10$ mg/L

① CL ?

$$CL = V_d \cdot K$$

$$\dot{q}(t) = -K\, q(t) + R$$

$$\dot{x}(t) = A\, x(t) + B\, u(t)$$

$t \to \infty \quad \bar{q}$

$$\bar{C} = \frac{\bar{q}}{V_d}$$

$x(t) = q(t) \qquad A = -K$

$u(t) = R \qquad B = 1$

$$\dot{q}(t) = 0 \qquad -K\bar{q} + R = 0$$

$$\bar{q} = \frac{R}{K} \Rightarrow \bar{C} = \frac{\bar{q}}{V_d} = \frac{R}{\underbrace{V_d \cdot K}_{CL}}$$

$$\bar{C} = \frac{R}{CL} \Rightarrow CL = \frac{R}{\bar{C}} = \frac{36}{10} = 3.6 \ [L/h]$$

② $R = 50$ mg/h

$$\dot{q}(t) = -K\, q(t) + R$$

$\bar{C} = ?$

$$\bar{C} = \frac{R}{CL} = \frac{50}{3.6} = 13.9 \ [mg/L]$$

③ Oral therapy.

$$\boxed{F = 0.9}$$

$\tau = 8$ [h]

$P_1 = 100$ mg $\qquad P_2 = 200$ mg

$P_3 = 300$ mg

$$\bar{C}_{ss} = 13.9 \ [mg/L]$$

$$\overline{C_{ss}} = \frac{R \cdot F}{V_d \cdot K \cdot \tau} = \frac{R F}{CL \cdot \tau}$$

$$R = \frac{\overline{C_{ss}} \cdot CL \cdot \tau}{F}$$

$$= \frac{13,9 \times 3,6 \cdot 8}{0.9} \approx \boxed{445 \ [mg]}$$

$R = 450$ mg

1 pill of 300 mg

1,5 pill of 100 mg

(4) $F = 0.8 \Rightarrow R = ?$ Therapy ?

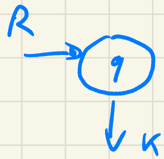
$$R = \frac{\overline{C_{ss}} \times CL \times \tau}{F} = \frac{13.9 \times 3,6 \times 8}{0.8} \approx 500 \text{ mg}$$

Therapy: 1 pill of 200 mg + 1 pill of 300 mg every 8 hours

EX 6 | $R = 100$ mg/h $\quad \tau_{1/2} = 15$ hr

$\bar{C} \in [20, 40]$ [mg/L] $\quad C(24) = 32$ mg/L

$\bar{C} = ?$



$\dot{q}(t) = -K\, q(t) + R$

$q(0) = 0$

$\bar{C} = \frac{\bar{q}}{Vol}$

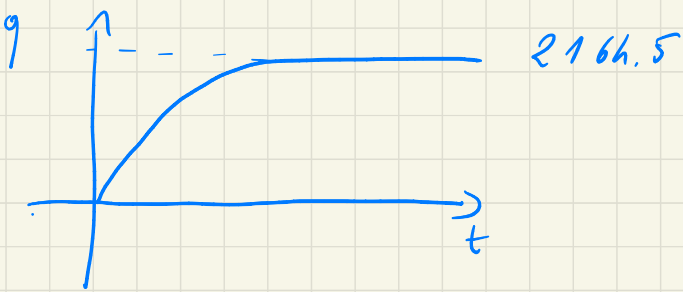
$\bar{q} \Rightarrow \dot{q}(t) = 0 \Rightarrow -K\bar{q} + R = 0$

$\bar{q} = \frac{R}{K}$

$t_{1/2} = 15$ h $\qquad t_{1/2} = \frac{\ln(2)}{K} \Rightarrow K = \frac{\ln(2)}{t_{1/2}}$

$K = \frac{0.693}{15} = 0,0462\ [h^{-1}]$

$\bar{q} = \frac{R}{K} = \frac{100}{0,0462} = \boxed{2164.5}$ [mg]



$\bar{C} = \frac{\bar{q}}{Vol}$

$C(t) = \frac{q(t)}{Vol}$

$q(0) = 0 \longrightarrow$ not usefull

$c(2h) = 32$ mg/h $\qquad q(2h) \rightarrow Vol = \dfrac{q(2h)}{c(2h)}$

$$\dot{q}(t) = -Kq(t) + R$$

$$\boxed{\begin{array}{l} \dot{x}(t) = Ax(t) + Bu(t) \\ x(t) = q \quad A = -K \quad B = 1 \\ u(t) = J = R \end{array}}$$

$$q(t) = \underbrace{q(0)\,e^{-Kt}}_{\text{Free move.}} + \underbrace{\int_0^t e^{-K(t-\tau)} R \, d\tau}_{\text{Forced move.}}$$

$$X(t) = \underbrace{X(0)\,e^{At}}_{\text{Free move.}} + \underbrace{\int_0^t e^{A(t-\tau)} B\,u(\tau)\,d\tau}_{\text{Forced movement}}$$

$$= R \int_0^t e^{-K(t-\tau)} \, d\tau$$

$$= \frac{R}{K} \left[e^{-K(t-\tau)} \right]_0^t$$

$$= \frac{R}{K} \left[1 - e^{-Kt} \right]$$

$$q(2h) = \frac{R}{K} \left(1 - e^{-K \times 2h} \right)$$

$$= \frac{100}{0.0462} \left(1 - e^{-0.0462 \times 24} \right)$$

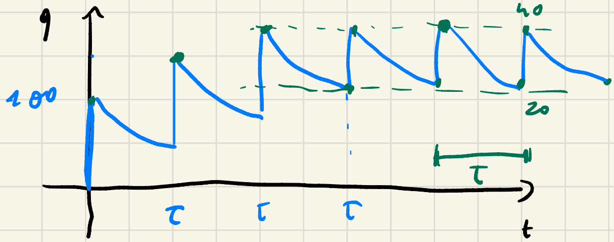
$$= 2164.5\,(1 - 0.33) = 1450.2 \text{ [mg]}$$

$$Vol = \frac{q(2h)}{c(2h)} = \frac{1450.2}{32} = 45.32 \text{ [L]}$$

$$\bar{c} = \frac{\bar{q}}{V_d} = \frac{2164.5}{45.32} = 47.76 \; [mg/L]$$

$$\bar{c} \in [20; 40]$$

(2) $\bar{c}_{ss}$? Multiple injections



$R = 100$ mg

Th. w. $[20; 40]$ mg/L

$t_{1/2} = 15$ h

$$\dot{q}(t) = -Kq(t) + R\,\delta(t) \Rightarrow \boxed{\dot{q}(t) = -Kq(t) \qquad q(0) = R}$$

$$q(j\tau^+) = R + q(j\tau^-) = R + \underline{q((j-1)\tau^+)\, e^{-K\tau}}$$

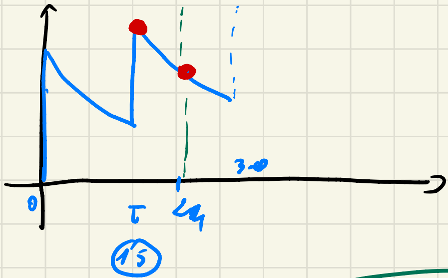
$K = 0.0462 \; [h^{-1}]$ $\qquad K = \frac{\ln(2)}{t_{1/2}}$

$\tau = t_{1/2} = 15$ h

$$\bar{c}_{ss} = \frac{R \cdot F}{V_d \cdot K \cdot \tau} \Rightarrow \begin{cases} R = 100 \\ F = 1 \\ K = 0.0462 \\ \tau = 15 \end{cases}$$

V_d? $\Rightarrow$ $V_d = \frac{q(24)}{c(24)}$ $\qquad c(24) = 32$ mg/L

$q(24) = ?$



$t = 2d$ is in between the second and the third injection.

$$\boxed{q(2d) = q(15^+)\, e^{-K\Delta t}} \qquad \Delta t = 2d - 15 = 9$$

$$= q(15^+)\, e^{-K \times 9}$$

$$q(15^+) = q(15^-) + R$$

$$q(15^-) = q(0)\, e^{-K \times 15} = 50 \text{ mg}$$

$t_{1/2} = 15$ h $\Rightarrow$ after 15 h $\Rightarrow$ $q(15^-) = \frac{q(0)}{2} = 50$ mg

$$q(15^+) = R + 50 = 150 \text{ mg}$$

$$q(2d) = q(15^+)\, e^{-K \times 9}$$

$$= 150\, e^{-0{,}0462 \times 9} = 98.97 \text{ mg}$$

$$Vol = \frac{q(2d)}{C(2d)} = \frac{98.97}{32} = 3.09 \text{ L}$$

$$\bar{C}_{ss} = \frac{R \cdot F}{Vol \cdot K \cdot \tau} = \frac{100 \cdot 1}{3.09 \times 0.0462 \times 15} \approx 46.7 \ [\text{mg}/L]$$

$\hat{R}$ s.t. $\bar{C}_{ss} = 40$

$$\hat{R} = \frac{\bar{C}_{ss} \cdot Vol \cdot K \cdot \tau}{F} = 40 \times 3.09 \times 0.0462 \cdot 15 = 85.7 \ [\text{mg}]$$

Ex 7 | $N = 10.000$ $I(0) = 100$

SIR

Time for contagion = 2 days

Duration of infection = 7 days

① $S(0)$? $I(0)$? $R(0)$?

$$\boxed{N = S(t) + I(t) + R(t)}$$

$$\begin{cases} I(0) = 100 \\ R(0) = 0 \\ S(0) = N - 100 = 9.900 \end{cases}$$

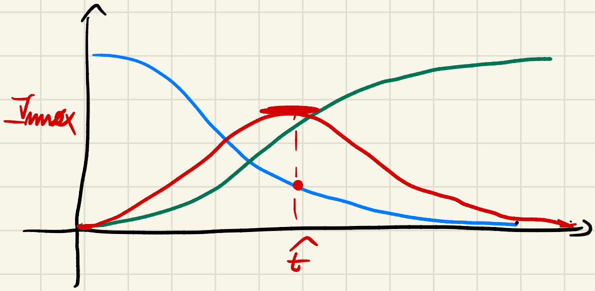
② $I_{max}(S) = ?$ $S(0) = 9.900$ $I(0) = 100$ $R = 0$

$$\begin{cases} \dot{S}(t) = -\dfrac{\beta S(t) I(t)}{N} \\ \dot{I}(t) = \dfrac{\beta S(t) I(t)}{N} - \gamma I(t) \\ \dot{R}(t) = \gamma I(t) \end{cases}$$

$\beta = \frac{1}{2}$

$\gamma = \frac{1}{7}$

$R_0 = \frac{\beta}{\gamma} = \frac{1/2}{1/7} = \frac{7}{2} = 3,5$



$\dot{I}(t) = 0 \Rightarrow \dfrac{\beta \bar{S} \bar{I}}{N} - \gamma \bar{I} = 0$

$$\left(\frac{\beta\bar{S}}{N}-\gamma\right)\bar{I}=0 \qquad \text{if } \bar{I}=0$$

$$\text{if } \left(\frac{\beta\bar{S}}{N}-\gamma\right)=0 \implies \boxed{\bar{S}=\frac{\gamma N}{\beta}}$$

$$\dot{S}(t)=\frac{dS}{dt} \qquad \dot{I}(t)=\frac{dI}{dt}$$

$$\frac{dS}{dt}=-\frac{\beta S I}{N} \qquad \frac{dI}{dt}=\frac{\beta S I}{N}-\gamma I$$

$$\frac{dI}{\cancel{dt}}\cdot\frac{\cancel{dt}}{dS}=\left(\frac{\beta SI}{N}-\gamma I\right)\cdot\left(-\frac{N}{\beta SI}\right)$$

$$\frac{dI}{dS}=\left(-1+\frac{\gamma N}{\beta S}\right)=\left(\frac{\gamma N}{\beta S}-1\right)$$

$$dI=\left(\frac{\gamma N}{\beta S}-1\right)dS$$

$$I=\int\left(\frac{\gamma N}{\beta S}-1\right)dS \qquad I \text{ as a funct of } S$$

$$=\frac{\gamma N}{\beta}\int\left(\frac{1}{S}-1\right)dS$$

$$=\frac{\gamma N}{\beta}\ln(S)-S+C$$

$t = 0$ $\quad I(0) = \frac{\gamma N}{\beta} \ln(S(0)) - S(0) + C$

$$C = \underbrace{I(0) + S(0)}_{N} - \frac{\gamma N}{\beta} \ln(S(0))$$

$$C = N - \frac{\gamma N}{\beta} \ln(S(0))$$

$$I = \frac{\gamma N}{\beta} \ln(S) - S + N - \frac{\gamma N}{\beta} \ln(S(0))$$

Solve for $I_{max} \Rightarrow \bar{S} = \frac{\gamma N}{\beta}$

$$I_{max} = \frac{\gamma N}{\beta} \ln\left(\frac{\gamma N}{\beta}\right) - \frac{\gamma N}{\beta} + N - \frac{\gamma N}{\beta} \ln(S(0))$$

$$\hat{=} 3592$$

$S = \frac{\gamma N}{\beta} = 2857$ $\qquad R = 10,000 - 3592 - 2,857$

$= 3551$

(3) $R(\infty)$? $\qquad R(\infty) = N$