EX 2

$$\begin{cases} x_1(t+1) = \frac{1}{2} x_1(t) + \alpha x_2(t) + v(t) \\ x_2(t+1) = \frac{1}{4} x_2(t) + \frac{1}{2} v(t) \\ y(t) = \frac{1}{2} x_1(t) \end{cases}$$

① $A = \begin{bmatrix} \frac{1}{2} & \alpha \\ 0 & \frac{1}{4} \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix}$ $C = \begin{bmatrix} \frac{1}{2} & 0 \end{bmatrix}$ $D = 0$

② $\lambda_{1,2} = \{ \frac{1}{2} ; \frac{1}{4} \}$ Both $|\lambda_i| < 1 \Rightarrow$ System A.S. $\forall \alpha$

③ $G(z) = C(zI - A)^{-1} B + D$

$$(zI - A) = \begin{bmatrix} z - \frac{1}{2} & -\alpha \\ 0 & z - \frac{1}{4} \end{bmatrix} \qquad (zI - A)^{-1} = \frac{1}{(z-\frac{1}{2})(z-\frac{1}{4})} \begin{bmatrix} z - \frac{1}{4} & \alpha \\ 0 & z - \frac{1}{2} \end{bmatrix}$$

$$G(z) = \frac{1}{(z-\frac{1}{2})(z-\frac{1}{4})} \begin{bmatrix} \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} z - \frac{1}{4} & \alpha \\ 0 & z - \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1/2 \end{bmatrix} + 0$$

$$= \frac{1}{(z-\frac{1}{2})(z-\frac{1}{4})} \begin{bmatrix} \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} z - \frac{1}{4} + \frac{\alpha}{2} \\ \frac{z}{2} - \frac{1}{4} \end{bmatrix} =$$

$$= \frac{\frac{z}{2} - \frac{1}{8} + \frac{\alpha}{4}}{(z-\frac{1}{2})(z-\frac{1}{4})}$$

④ $N = \underline{1} \Rightarrow G(1) = \underline{1}$

$$G(1) = \frac{\frac{1}{2} - \frac{1}{8} + \frac{\alpha}{4}}{\left(1-\frac{1}{2}\right)\left(1-\frac{1}{4}\right)} = \frac{\frac{3}{8} + \frac{\alpha}{4}}{\frac{1}{2}\cdot\frac{3}{4}} = \frac{\frac{3+2\alpha}{\cancel{8}}}{\frac{3}{\cancel{8}}} =$$

$$= \frac{3+2\alpha}{3} = 1 + 2\alpha$$

$$G(1) = 1 \Rightarrow \cancel{1} + 2\alpha = \cancel{1} \Rightarrow \boxed{\alpha = 0}$$

⑤ $\alpha = 1$

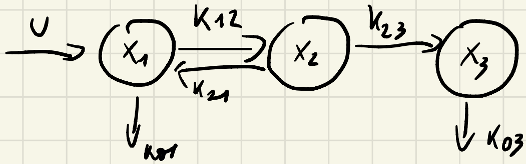
$$U(t) = \begin{cases} 0 , & t \leq 0 \\ 3 , & t > 0 \end{cases}$$

$$\alpha = 1 \Rightarrow \quad G(z) = \frac{\frac{z}{2} - \frac{1}{8} + \frac{1}{4}}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)} = \frac{\frac{z}{2} + \frac{1}{8}}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}$$

$$G(1) = \frac{\frac{1}{2} + \frac{1}{8}}{\left(1-\frac{1}{2}\right)\cdot\left(1-\frac{1}{4}\right)} = \frac{\frac{5}{8}}{\frac{1}{2}\cdot\frac{3}{4}} = \frac{\frac{5}{\cancel{8}}}{\frac{3}{\cancel{8}}} = \frac{5}{3}$$

$$\bar{y} = N \cdot \bar{U} \qquad \Rightarrow \bar{y} = \frac{5}{3} \cdot 3 = \boxed{5}$$

Ex 4



① State-space model:

$$\begin{cases} \dot{x}_1(t) = -(K_{01}+K_{12})\,x_1(t) + K_{21}\,x_2(t) + u(t) \\ \dot{x}_2(t) = K_{12}\,x_1(t) - (K_{21}+K_{23})\,x_2(t) \\ \dot{x}_3(t) = K_{23}\,x_2(t) - K_{03}\,x_3(t) \\ y(t) = x_1(t) \end{cases}$$

$$A = \begin{bmatrix} -(K_{01}+K_{12}) & K_{21} & 0 \\ K_{12} & -(K_{21}+K_{23}) & 0 \\ 0 & K_{23} & -K_{03} \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \quad D = 0$$

② $K_{12} = 0.5$, $K_{21} = 0.25$ $K_{23} = 0.1$ $K_{01} = K_{03} = 0.05$

$$A = \begin{bmatrix} -0.55 & 0.25 & 0 \\ 0.5 & -0.35 & 0 \\ 0 & 0.1 & -0.05 \end{bmatrix}$$

1. $A_{ii} \leq 0$ ✓

2. $A_{ji} \geq 0$, $j \neq i$ ✓

3. $|A_{11}| = 0.55 > \sum_{j=1, j\neq 1}^{3} A_{j1} = 0.5$ ✓

$|A_{22}| = 0.35 = \sum_{j=1, j\neq 2}^{3} A_{j2} = 0.35$ ✓

$$|A_{33}| = 0{,}05 > \sum_{\substack{j=1\\ j\neq 3}}^{3} A_{j3} = 0 \quad \checkmark$$

The system is ASYMP. STABLE

③ $\bar{M} = 2$ $\qquad \bar{M} = \bar{x}_1 + \bar{x}_2 + \bar{x}_3$

$$\begin{cases} -0.55\,\bar{x}_1 + 0.25\,\bar{x}_2 + \bar{u} = 0 \\ 0.5\,\bar{x}_1 - 0.35\,\bar{x}_2 = 0 \\ 0.1\,\bar{x}_2 - 0.05\,\bar{x}_3 = 0 \end{cases}$$

$\Rightarrow \bar{x}_1 = \dfrac{0{,}35}{0{,}5}\,\bar{x}_2 = 0.7\,\bar{x}_2$

$\Rightarrow \bar{x}_3 = \dfrac{0.1}{0{,}05}\,\bar{x}_2 = 2\,\bar{x}_2$

$\bar{x}_1 + \bar{x}_2 + \bar{x}_3 = 2 \Rightarrow 0.7\,\bar{x}_2 + \bar{x}_2 + 2\bar{x}_2 = 2$

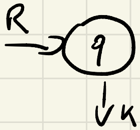
$3{,}7\,\bar{x}_2 = 2 \Rightarrow \bar{x}_2 = 0.54$

$\bar{x}_1 = 0.38$

$\bar{x}_3 = 1{,}08$

$$\bar{u} = 0{,}55\,\bar{x}_1 - 0{,}25\,\bar{x}_2 = 0.209 - 0.135 = \boxed{0{,}074} \quad \checkmark$$

EX 6



$R = 300$ mg $\qquad K = 0.15\ [h^{-1}]$

$c(3.5) = 88.75\ [mg/L]$

$$\dot{q}(t) = -Kq(t) + R\,\delta(t) \Rightarrow \dot{q}(t) = -Kq(t) \qquad q(0) = R$$

① $Vd = ?$ $\quad CL = ?$

$$Vd = \frac{q(t)}{c(t)}$$

I have $c(3.5)$

↳ let's compute $q(3.5)$

$$q(t) = q(0)\, e^{-K \times t} \Rightarrow q(3.5) = 300\, e^{-0.15 \times 3.5} = 177.47\ [mg]$$

$$Vd = \frac{q(3.5)}{c(3.5)} = \frac{177.47}{88.75} \approx 2\ [L]$$

$$CL = Vd \cdot K = 2 \cdot 0.15 = 0.3\ [L/h]$$

② $c(2)$?

$$c(2) = \frac{q(2)}{Vd} = \frac{q(0)}{Vd}\, e^{-K \times 2} = c(0)\, e^{-K \times 2}$$

$$= 150\, e^{-0.3} = 111.12\ [mg/L]$$

③ $\hat{t}$ s.t. $c(\hat{t}) = 50\ [mg/L]$

$$c(\hat{t}) = c(0)\, e^{-K\hat{t}} \Rightarrow e^{-K\hat{t}} = \frac{c(\hat{t})}{c(0)}$$

$$-K\hat{t} = \ln\left(\frac{c(\hat{t})}{c(0)}\right)$$

$$\Rightarrow \hat{t} = \frac{\ln\left(\frac{c(\hat{t})}{c(0)}\right)}{-K} = 7.32 \ [h]$$