UNIVERSITÀ DEGLI STUDI DI BERGAMO

Dipartimento di Ingegneria Gestionale, dell'Informazione e della Produzione

Control Automation Lab

ADAPTIVE LEARNING, ESTIMATION AND SUPERVISION OF DYNAMICAL SYSTEMS (ALES)

Lecture 1: Introduction

Master Degree in COMPUTER ENGINEERING

Data Science and Data Engineering Curriculum

SPEAKER
Prof. Mirko Mazzoleni

PLACE
University of Bergamo

Who I am

- **Name:** Mirko Mazzoleni

- **Currently:** Assistant Professor (RTD-B), **Control and Automation Lab (CAL UniBG)**

✓ *Research:* System identification, machine learning, fault diagnosis

✓ *Teaching:*
1. Identificazione dei Modelli e Analisi dei Dati (IMAD)
2. Adaptive learning, estimation and supervision of dynamical systems (ALES)
3. Sustainable and Industrial System laboratory

- **Other:** Co-founder of AISent srl startup https://aisent.io/ AISENT

- **Contacts**

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✓ https://mirkomazzoleni.github.io/

✓ http://cal.unibg.it/ **Website CAL UniBG**

✓ https://www.facebook.com/ControlAutomationLabUnibg/

Control and Automation Laboratory (CAL) @ University of Bergamo, Italy

People:

- 3 professors
- 1-3 visiting professors
- 6 PhD students

- **Teaching** (control systems)
- **Research** (advanced control, optimization, fault diagnosis, identification)
- **Industrial projects** (manufacturing, aerospace, packaging,...)

Outline

1. Presentation of the Adaptive Learning, Estimation and Supervision of dynamical systems (ALES) course

2. What we have learnt so far…

3. …and what we still have to learn!

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1. **Presentation of the Adaptive Learning, Estimation and Supervision of dynamical systems (ALES) course**

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Prerequisites for the course

It is **strongly suggested to have** good foundations in the following topics:

- Systems identification (IMAD)
- Control systems
- Linear algebra
- Calculus 1 and 2

How to «refresh» the prerequisites?

- Systems identification (IMAD) ⇒ Unibg course
- Control systems ⇒ Unibg courses
- Linear algebra ⇒ Unibg course, Gilbert Strang course @MIT on Youtube
- Calculus I and II ⇒ UniBg course

Exam

- Short **written** exam **1 hour + coding project**
- **Open questions** about the theory
- **Matlab implementation** of a technique\algorithm from the literature + oral power point presentation

How to be prepared for the exam?

- Follow the lessons
- Study the theory
- Implement by yourself the algorithms presented at lesson

Educational objectives of the course

At the end of the course, you will be able to:

- **Employ** *recursive* and *adaptive estimation methods*, without and with constraints on the estimation variables

- **Identify** models of dynamical systems in *state-space form*, with *multiple-inputs and multiple-outputs* (MIMO), and in *closed-loop* settings

- **Define** and **solve** industrial *fault diagnosis problems*, identify their main components and envisage a possibile solution using different approaches

Teaching materials

Materials provided by the teacher

- Lectures slides
- **Sorry. No code** ☹*:* you have to implement the algorithms by your own MATLAB®

Interaction and feedback

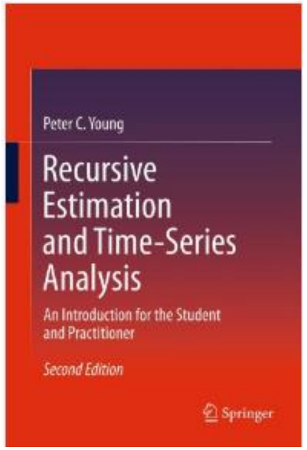
- During the week I will give you **activities** to do and **tests** to answer. They are **optional** but they help you to understand the degree of learning before the exam. In addition, they will contribute to giving a bonus of **+3 points** to the final grade

We will use **MS Teams activities**

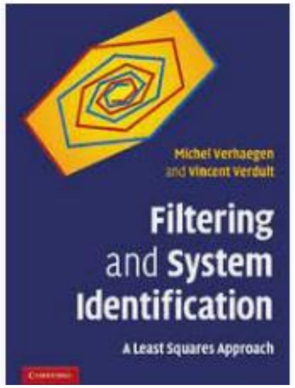
Teaching materials

Suggested textbooks

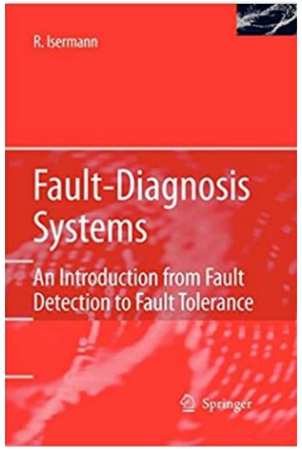
- Peter C. Young, **Recursive Estimation and Time-Series Analysis**, 2° ed. Springer (2011)



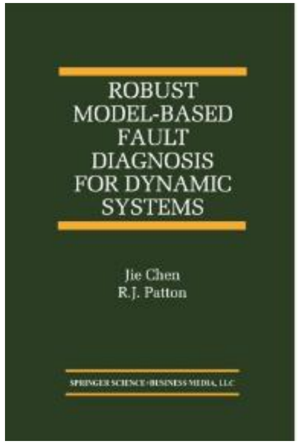
- Michel Verhaegen, **Filtering and system identification: a least squares approach**, Cambridge University Press (2007)



- Rolf Isermann, **Fault-Diagnosis Systems: An Introduction From Fault Detection To Fault Tolerance**, Springer (2005)



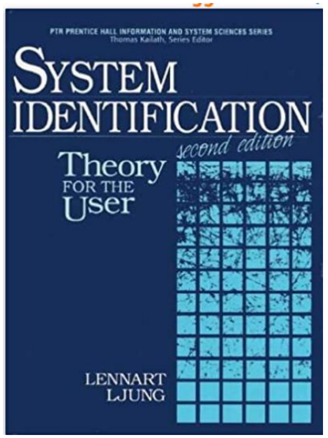
- Jie Chen, Ron J. Patton, **Robust Model-Based Fault Diagnosis for Dynamic Systems**, Springer (1999)



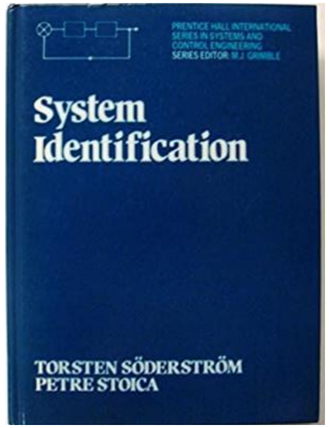
Teaching materials

Other suggested textbooks

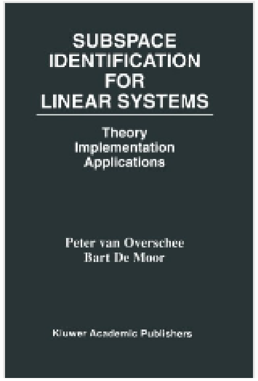
- Lennart Ljung, **System Identification: Theory for the User**, Pearson (1998)



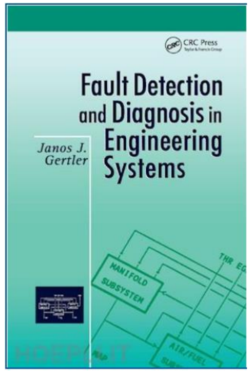
- Torsten Soderstrom, Petre Stoica, **System identification** Prentice Hall international (2001)



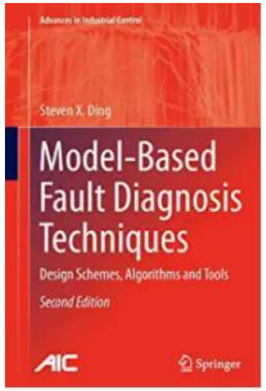
- Peter Van Overschee, Bart De Moor, **Subspace identification for linear systems**, Springer (1996)



- Janos Gertler, **Fault Detection and Diagnosis in Engineering Systems**, CRC Press, (1998)



- Steven X. Ding, **Model-baed fault diagnosis techniques**, 2° ed, Springer (2013)



Syllabus

1. **Recursive and adaptive identification**
   1.1 Recursive ARX estimation (RLS)
   1.2 Least Mean Squares (LMS)
   1.3 Instrumental Variables (IV)

2. **Subspace and MIMO identification**
   2.1 Singular Value Decomposition
   2.2 Impulse data: Ho-Kalman, Kung algorithms
   2.3 Generic I/O data: the MOESP algorithm

3. **Closed-loop identification**

4. **Supervision of dynamical systems**
   4.1 Introduction to fault diagnosis
   4.2 Model-based fault diagnosis
   4.3 Parity space approaches
   4.4 Observer-based approaches
   4.5 Signal-based fault diagnosis
   4.6 Knowledge-based fault diagnosis

CASE STUDIES

- Virtual Reference Feedback Tuning
- Nuclear particles classification
- Leak detection in an industrial valve
- Bearing fault identification

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1. Presentation of the Adaptive Learning, Estimation and Supervision of dynamical systems (ALES) course

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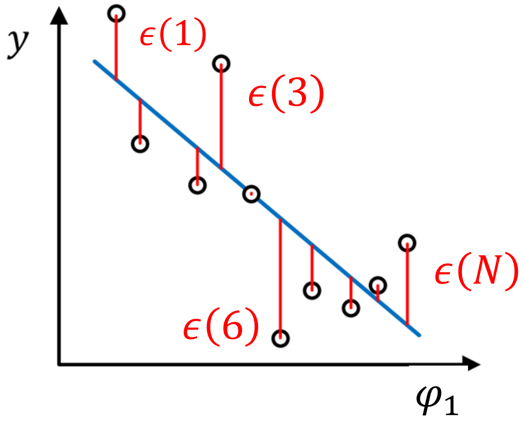
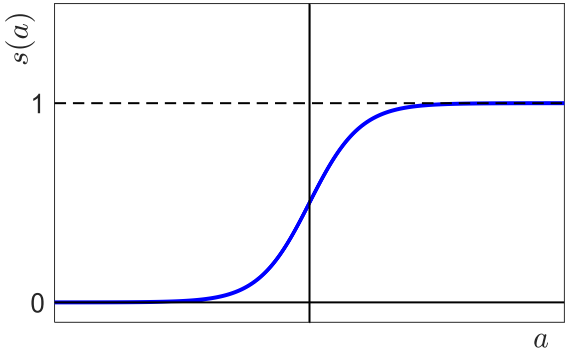
Modeling static systems and machine learning

Linear regression model



$$y(i) = \boldsymbol{\varphi}^\top(i)\boldsymbol{\theta} + \epsilon(i)$$

Logistic regression model



$$P(y = 1|\boldsymbol{\varphi}) = \frac{1}{1 + e^{-\boldsymbol{\varphi}^\top\boldsymbol{\theta}}}$$

Linear regression algorithm

$$J(\boldsymbol{\theta}) = \frac{1}{N}(X\boldsymbol{\theta} - Y)^\top(X\boldsymbol{\theta} - Y)$$

$$\hat{\boldsymbol{\theta}} = (X^\top X)^{-1}X^\top Y$$

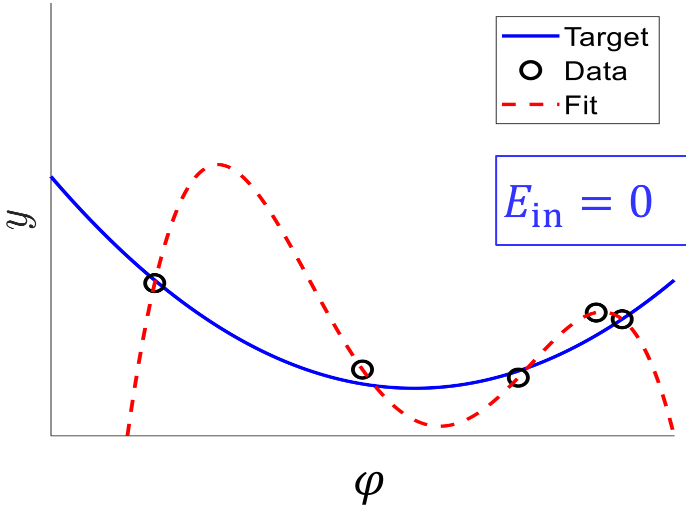
Logistic regression algorithm

$$J(\boldsymbol{\theta}) = -\sum_{i=1}^{N}\left(y(i)\ln\pi(i) + (1 - y(i)\ln[1 - \pi(i)]\right)$$

Gradient descent $\hat{\boldsymbol{\theta}}^{(k+1)} = \hat{\boldsymbol{\theta}}^{(k)} - \alpha \cdot \nabla J(\boldsymbol{\theta})\Big|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}^{(k)}}$

Modeling static systems and machine learning

Overfitting

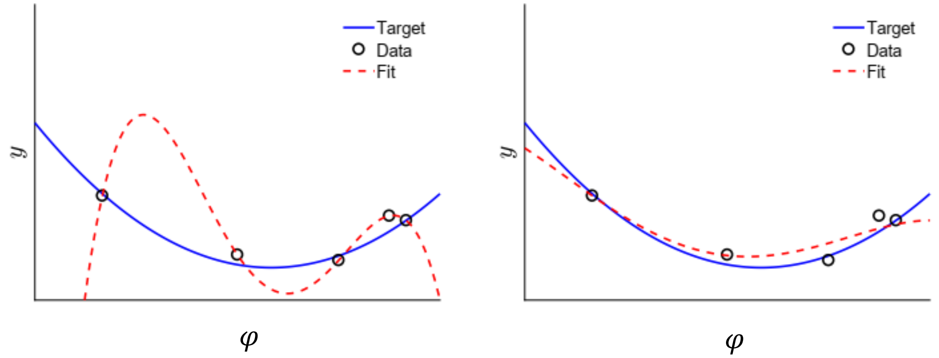


$E_{\text{in}} = 0 \qquad E_{\text{out}} = \textbf{huge}$

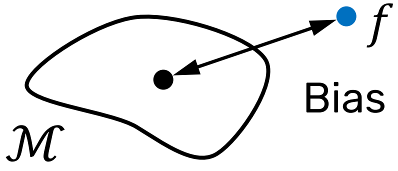
Regularization

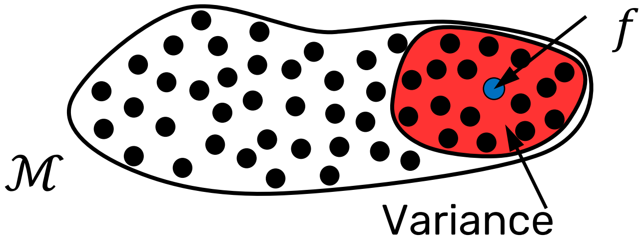
$$E_{\text{aug}}(\boldsymbol{\theta}) = \frac{1}{N}\sum_{i=1}^{N}\left(y(i) - h(\boldsymbol{\varphi}(i);\,\boldsymbol{\theta})\right)^2 + \lambda_{\text{reg}} \cdot \sum_{j=0}^{d-1}\left(\theta_j\right)^2$$

λ_{reg_1} $\qquad$ $\lambda_{\text{reg}_2} > \lambda_{\text{reg}_1}$



Bias and variance



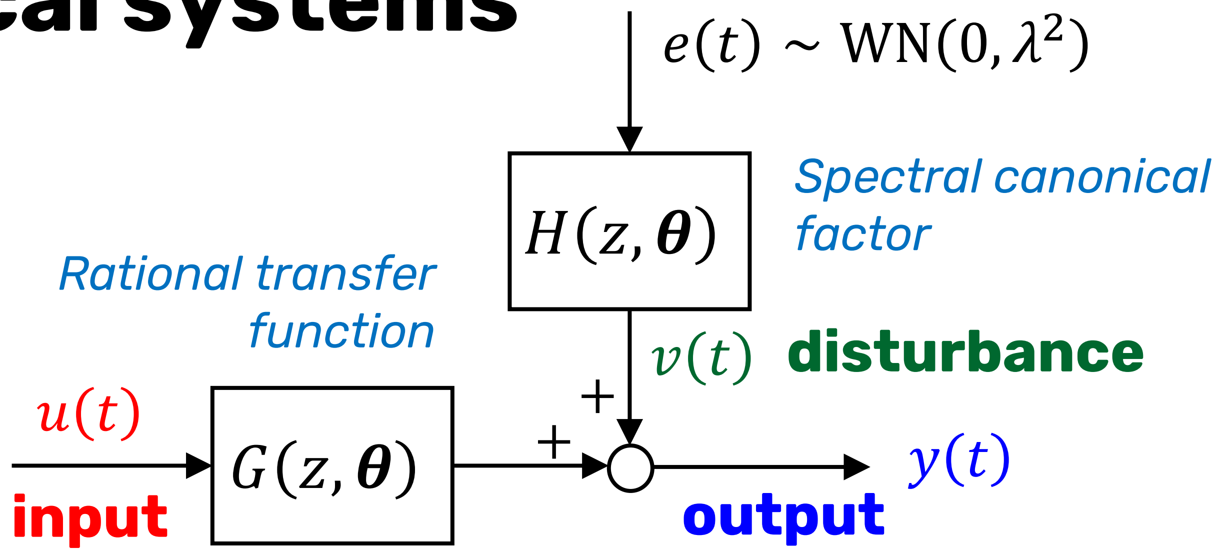


Validation and cross-validation

Train (60%)	Validation (20%)	Test (20%)

Modeling LTI SISO dynamical systems

In the IMAD course, we studied **Prediction Error Methods** (PEM) for **SISO LTI** dynamical systems



The **general form** of the models was:

- $e(t)$ is **a white noise** signal, introduced to account for all **unmodeled dynamics** and the effects of **external unmeasured disturbances**

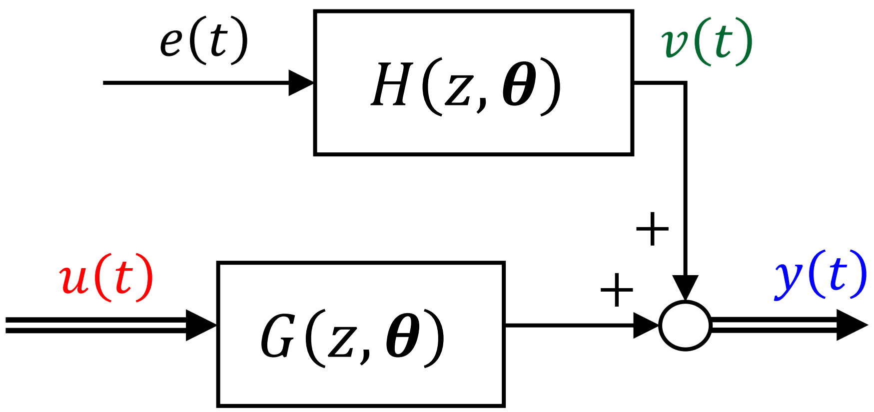
- $G(z; \boldsymbol{\theta})$ and $H(z; \boldsymbol{\theta})$ are suitable **transfer functions** that depend on parameters $\boldsymbol{\theta}$

$$\mathcal{M}(\boldsymbol{\theta}): y(t) = G(z, \boldsymbol{\theta})u(t-1) + H(z, \boldsymbol{\theta})e(t), \qquad e(t) \sim \mathrm{WN}(0, \lambda^2)$$

Modeling LTI SISO dynamical systems

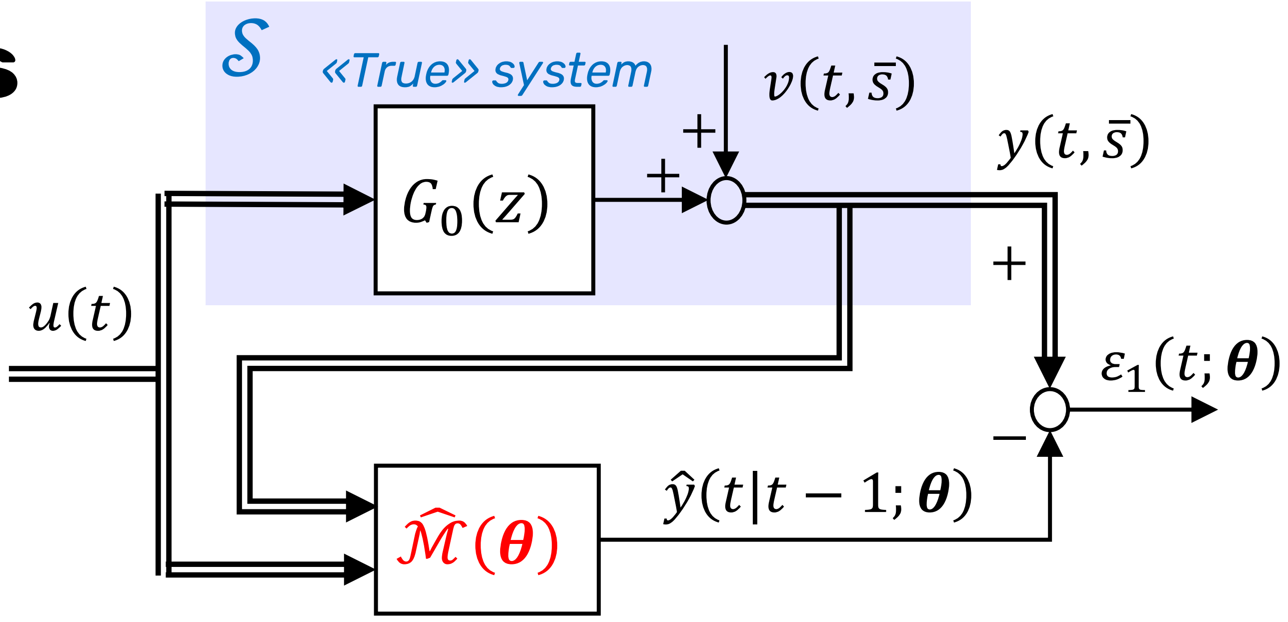
Family of models	$G(z,\boldsymbol{\theta})$	$H(z,\boldsymbol{\theta})$
ARX	$\frac{B(z,\boldsymbol{\theta})}{A(z,\boldsymbol{\theta})} z^{-k}$	$\frac{1}{A(z,\boldsymbol{\theta})}$
ARMAX	$\frac{B(z,\boldsymbol{\theta})}{A(z,\boldsymbol{\theta})} z^{-k}$	$\frac{C(z,\boldsymbol{\theta})}{A(z,\boldsymbol{\theta})}$
OE	$\frac{B(z,\boldsymbol{\theta})}{F(z,\boldsymbol{\theta})} z^{-k}$	1
FIR	$B(z,\boldsymbol{\theta}) z^{-k}$	1
BJ	$\frac{B(z,\boldsymbol{\theta})}{F(z,\boldsymbol{\theta})} z^{-k}$	$\frac{C(z,\boldsymbol{\theta})}{D(z,\boldsymbol{\theta})}$

The $\boldsymbol{\theta}$ vector represents the **model parameters** (value of the coefficients of the $A(z), B(z), C(z)$ polynomials)



Prediction Error Methods

Given a model family $\mathcal{M}(\boldsymbol{\theta})$, the estimate of $\boldsymbol{\theta}$ was performed by minimizing variance of the **one-step prediction error** $\varepsilon_1(t;\boldsymbol{\theta})$



Model in «prediction form»

Model in prediction form

$$\widehat{\mathcal{M}}(\boldsymbol{\theta}):\ \hat{y}(t|t-1;\boldsymbol{\theta}) = H^{-1}(z,\boldsymbol{\theta})G(z,\boldsymbol{\theta})\cdot u(t) + [1 - H^{-1}(z,\boldsymbol{\theta})]\cdot y(t)$$

PEM estimate

$$\varepsilon_1(t;\boldsymbol{\theta}) = y(t) - \hat{y}(t|t-1;\boldsymbol{\theta})$$

$$J_N(\boldsymbol{\theta}) = \frac{1}{N}\sum_{t=1}^{N}\varepsilon_1(t;\boldsymbol{\theta})^2 \qquad \widehat{\boldsymbol{\theta}}_N = \arg\min_{\boldsymbol{\theta}\in\Theta} J_N(\boldsymbol{\theta})$$

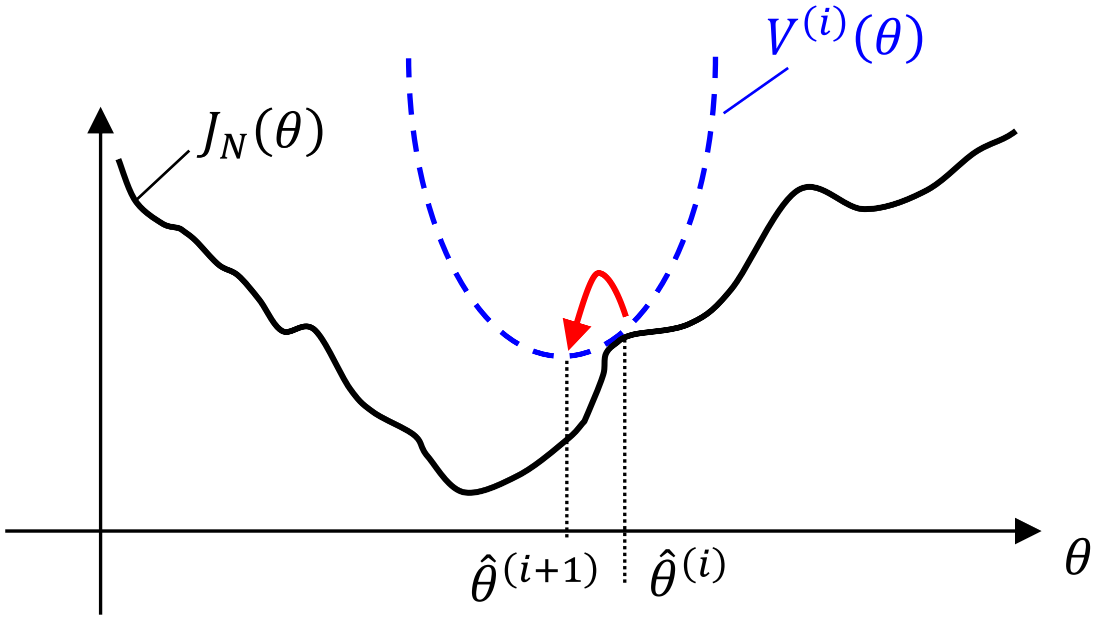
Prediction Error Methods: estimation algorithms

- **ARX, FIR: convex** cost function (global minimum) ⇨ **Least squares**
- **ARMAX, BJ, OE: not convex** cost function (local minima) ⇨ **Quasi-Newton iterative schemes**

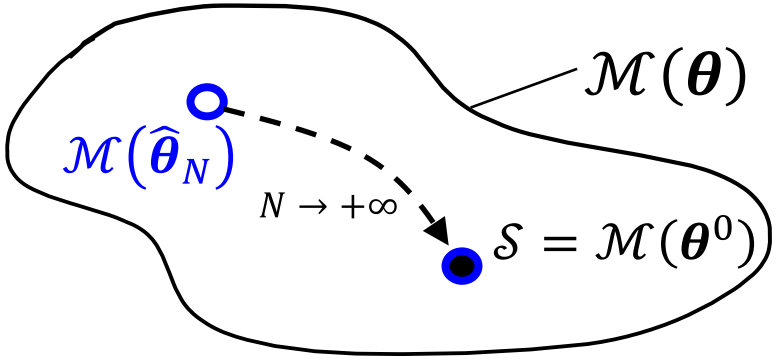
ARX, FIR

$$\hat{\boldsymbol{\theta}}_N = \left[\sum_{t=1}^{N} \boldsymbol{\varphi}(t)\boldsymbol{\varphi}^\top(t)\right]^{-1} \cdot \left[\sum_{t=1}^{N} \boldsymbol{\varphi}(t)y(t)\right]$$

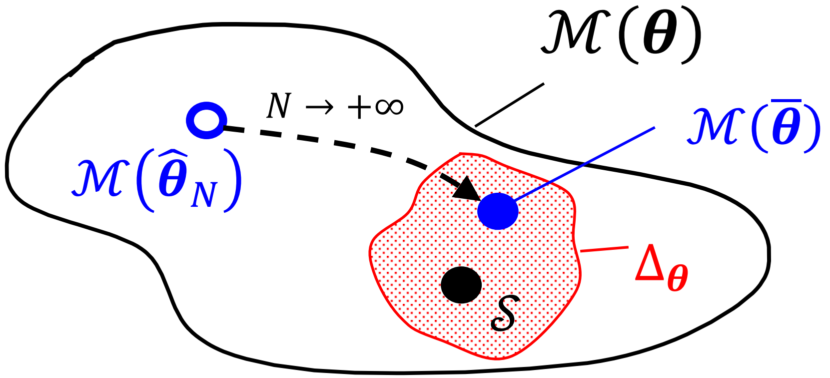
ARMAX, BJ, OE



Prediction Error Methods: asymptotic properties



$$\hat{\boldsymbol{\theta}}_N \xrightarrow[N \to +\infty]{} \bar{\boldsymbol{\theta}} = \boldsymbol{\theta}^0$$



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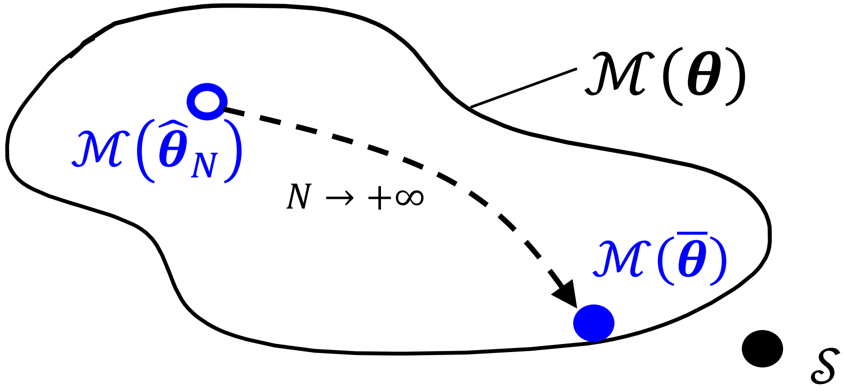
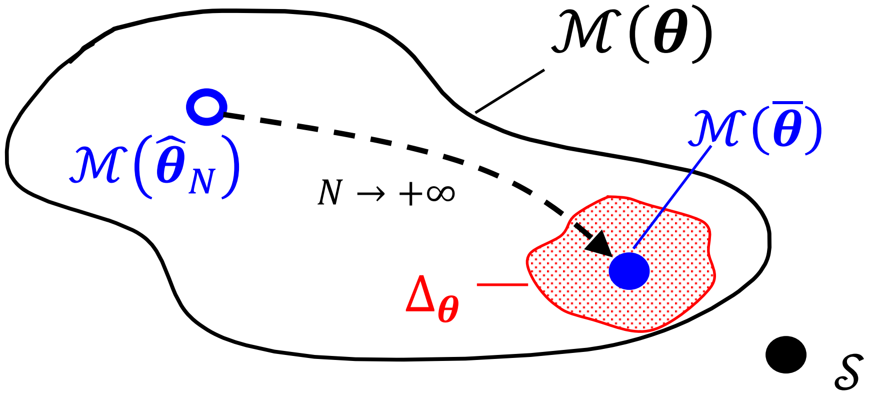


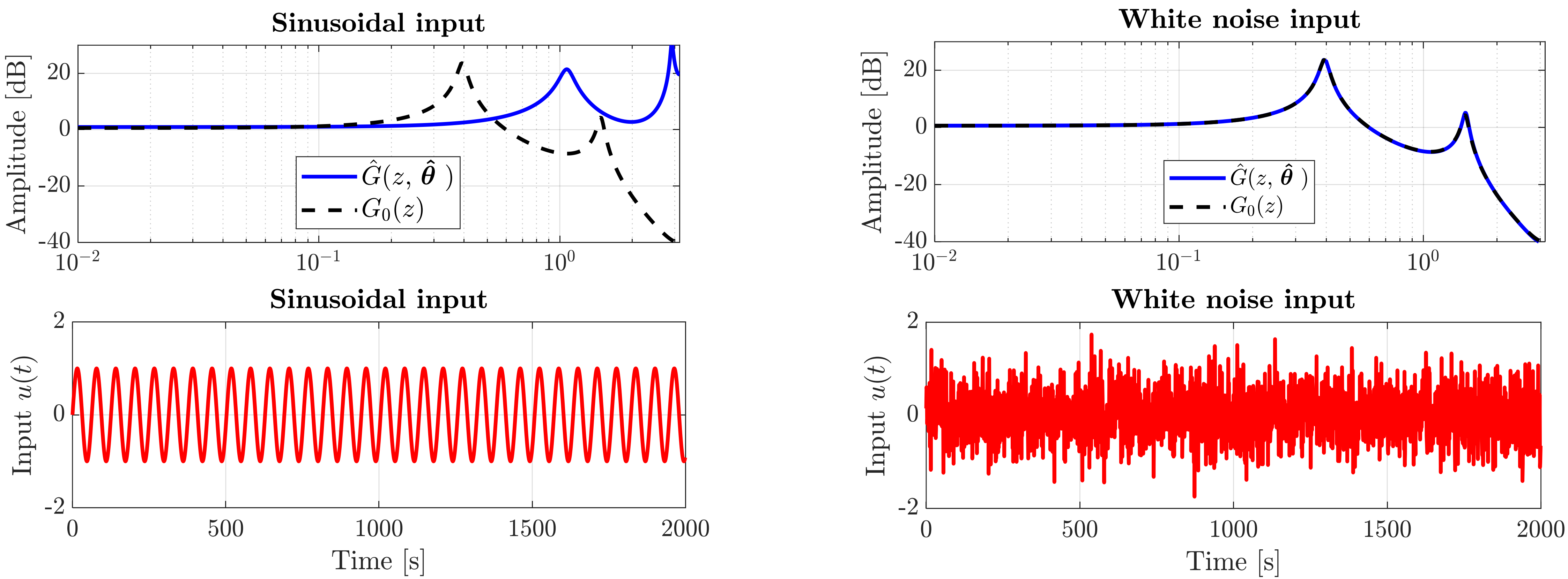
$$\hat{\boldsymbol{\theta}}_N \xrightarrow[N \to +\infty]{} \bar{\boldsymbol{\theta}} \neq \boldsymbol{\theta}^0$$



$$\hat{\boldsymbol{\theta}}_N \xrightarrow[N \to +\infty]{} \bar{\boldsymbol{\theta}} \neq \boldsymbol{\theta}^0$$

Prediction Error Methods: persistent excitation

Experimental identifiability requires the input signal to be **persistently exciting** of enough order, as **white noise, Pseudo-Random Binary Signal (PRBS), Multisine**

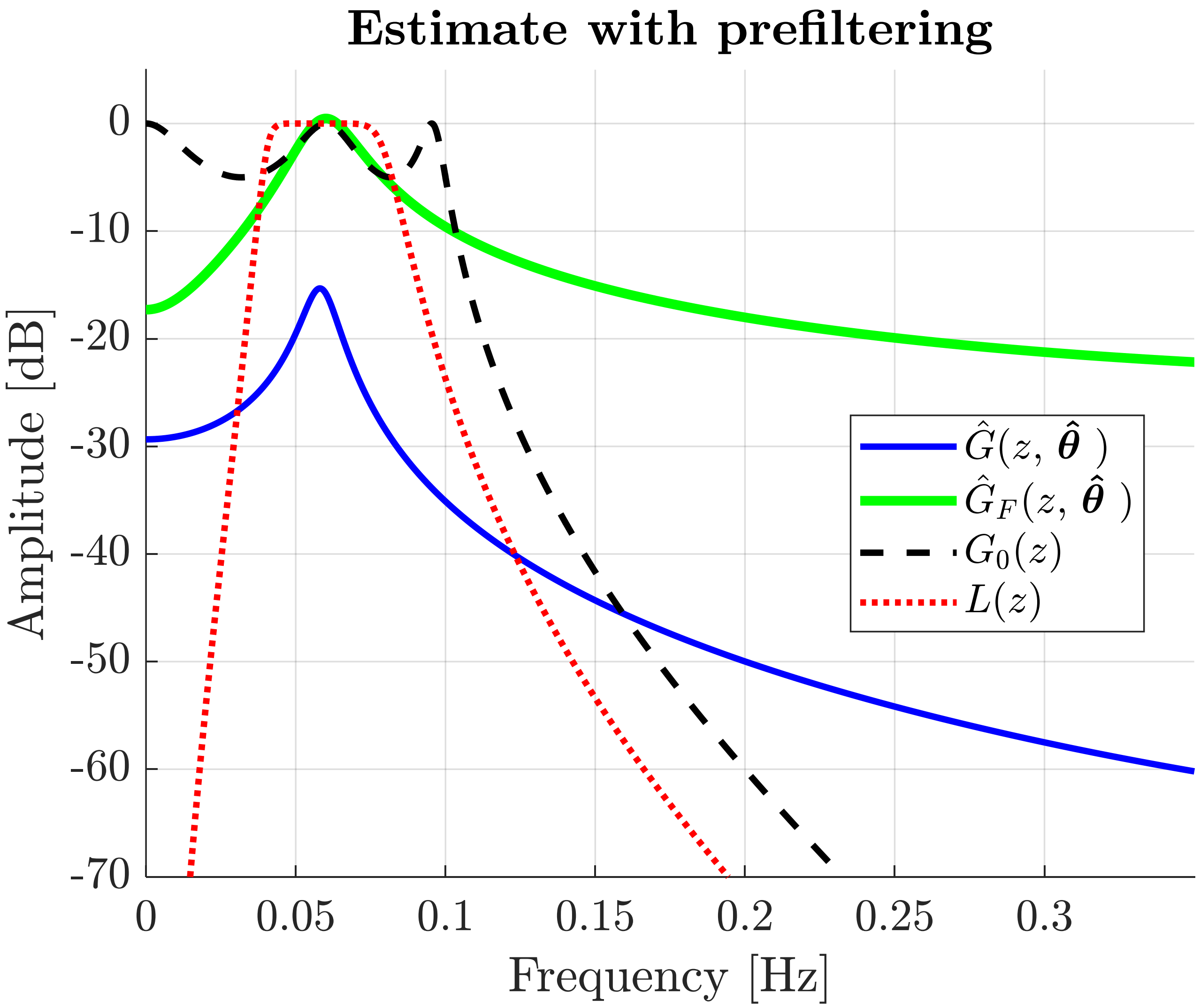


Prediction Error Methods: robustness

When the chosen model is not able to exactly represent the true system, we can **prefilter the data** to perform **bias-shaping**

Focus the modelling efforts in a selected bandwidth

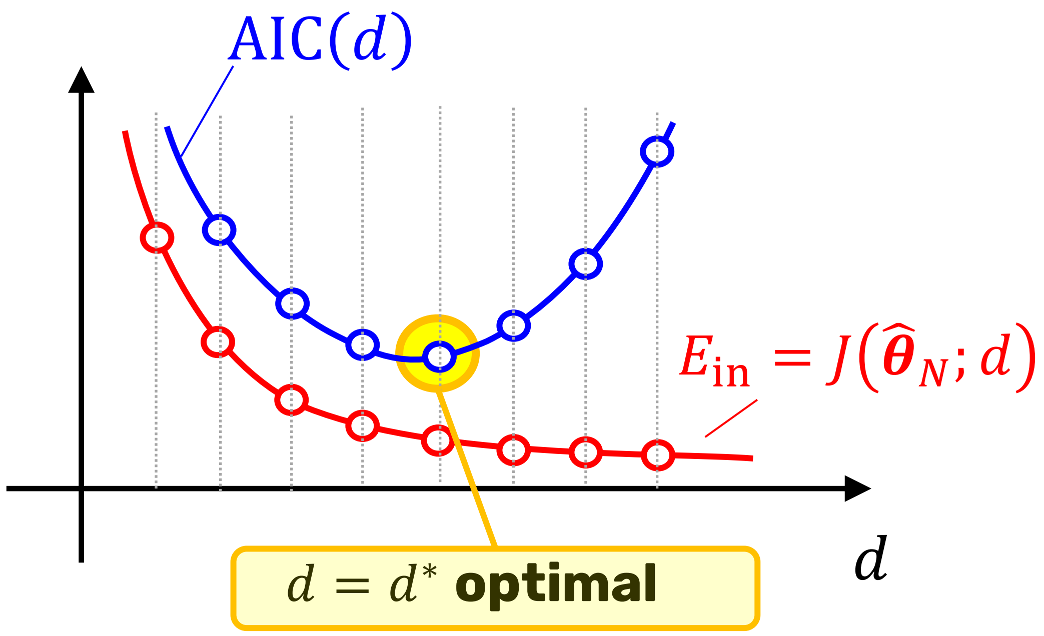
$$\bar{J}(\boldsymbol{\theta}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left|\Delta G\left(e^{j\omega}, \boldsymbol{\theta}\right)\right|^2 \cdot g_F(\omega, \boldsymbol{\theta})\, d\omega$$



Prediction Error Methods: validation techniques

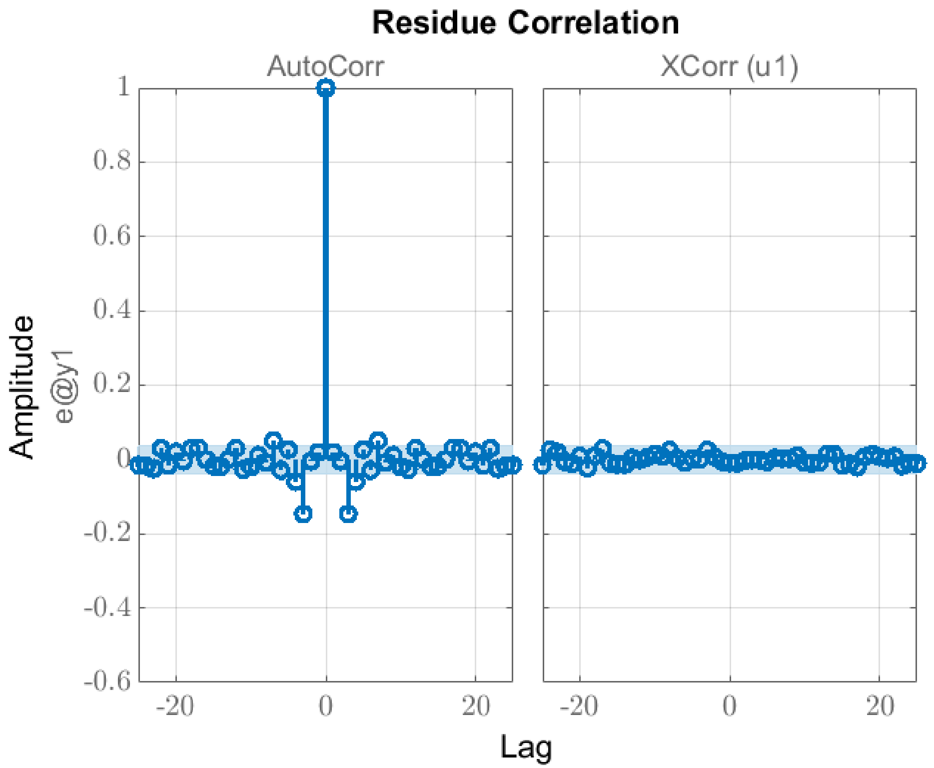
Choosing the model complexity

- Complexity formulas
- Validation



Assessing model goodness

- Residual analysis
- Uncertainty of the estimates
- Prediction and simulation fitness



Empirical Transfer Function Estimate (ETFE)

Suppose we divide the signals $u(t)$ and $y(t)$, both multisines, into sub-sequences $u^{[p]}, y^{[p]}$ which contain the different $p = 1, \dots, P$. We define:

$\breve{U}^{[p]} = \mathrm{DFT}\left(u^{[p]}\right)$ **input DFT**

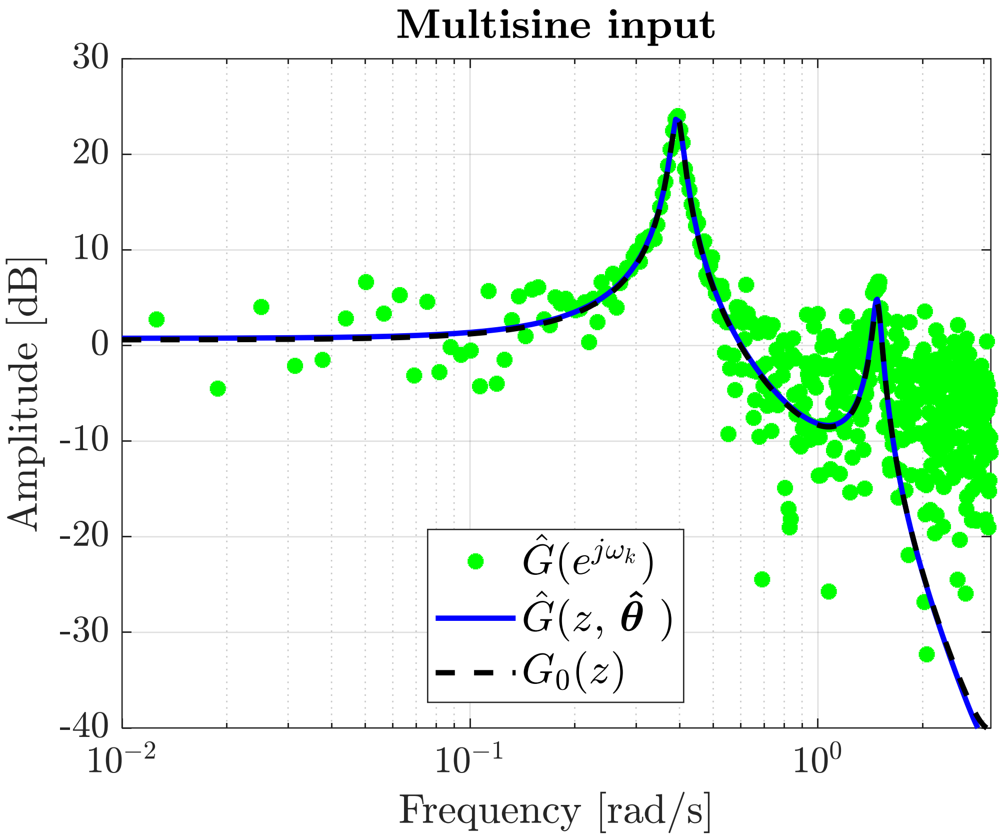
$\breve{Y}^{[p]} = \mathrm{DFT}\left(y^{[p]}\right)$ **output DFT**

$$\widehat{U}(k) = \frac{1}{P} \sum_{p=1}^{P} \breve{U}^{[p]}(k)$$

$$\widehat{Y}(k) = \frac{1}{P} \sum_{p=1}^{P} \breve{U}^{[p]}(k)$$

$$\Rightarrow \quad \widehat{G}(k) = \frac{\widehat{Y}(k)}{\widehat{U}(k)}$$

Empirical Transfer Function Estimate (ETFE)



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Adaptive Learning, Estimation and Supervision of dynamical systems

A) ADAPTIVE LEARNING

The identification algorithms we have learnt so far process all available data as a **single batch.** This approach has several drawbacks:

- **memory occupancy:** we need to store all data in the computing device
- **computational load:** we may need to perform a big matrix inversion
- **time delay:** if we need to take quick decisions, an immediately available - albeit approximate – model, may be more useful than a perfect model available only after a batch of data has been collected

Adaptive Learning, Estimation and Supervision of dynamical systems

In many applications data are not available all together, but they are **collected one by one on-line**. Can we still identify a model in these conditions?

RECURSIVE IDENTIFICATION METHODS

Sometimes the system generating the data is **not time-invariant** (or it is nonlinear).

Can we recompute the model **on-line** adapting it to the current behavior of the system?

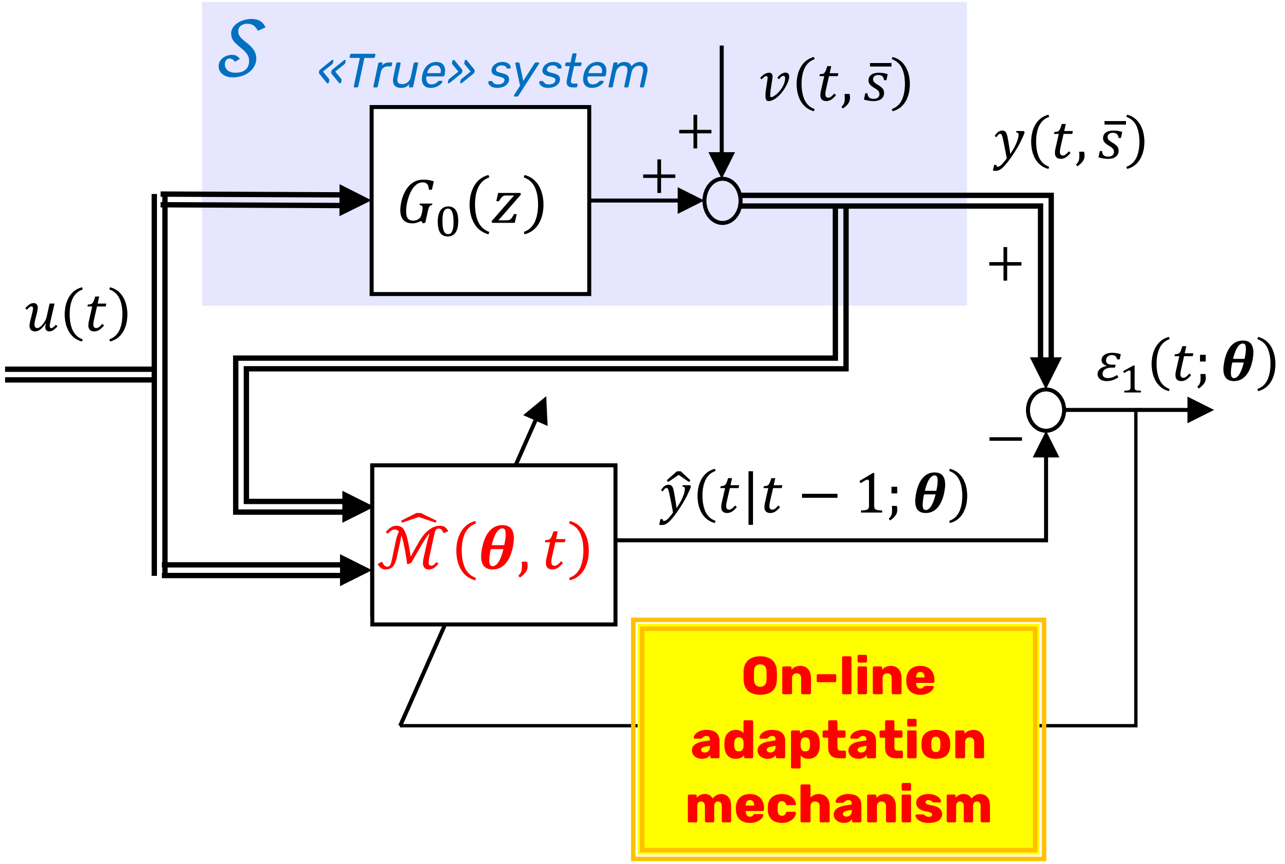
ADAPTIVE IDENTIFICATION METHODS

Adaptive Learning, Estimation and Supervision of dynamical systems

When a model is:

- time-varying, or
- Nonlinear but linearly approximated

the linear model has only **local validity** and must be **online updated** to the most recent data available *(model tracking)*



Adaptive Learning, Estimation and Supervision of dynamical systems

The main applications of **recursive and adaptive learning methods** are:

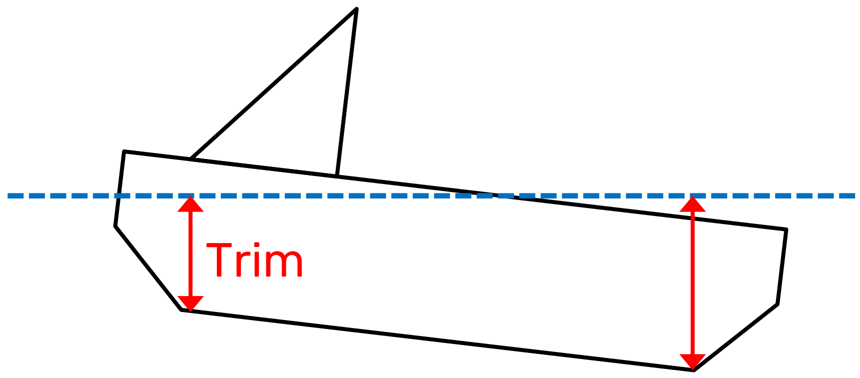
1. **Adaptive filtering and control**: a time-varying controller or filter is tuned to the **currently estimated** model of the process *(iterative learning control, extremum-seeking control, model reference adaptive control, adaptive filtering and equalization)*

2. **Fault diagnosis**: a model of the system is continuously updated to rapidly identify significant **changes in its parameters**. For LTI systems, a change in a parameter might be a symptom of a system degradation due to a fault *(diagnosis of parametric faults)*

Example: ship steering adaptive control

A **ship's heading angle** and position is controlled using the **rudder angle**. For large ships, its response to a change in rudder angle is so **slow** that it is affected by random components as **wind** and **wave motion**

The steering dynamics of the ship depends on its shape, size, loading, trim, and water depth. In particular, **loading** and **water depth** may **vary** with time



The rudder angle controller must be **continuously retuned** to match the dynamics of the system

Example: self-tuning predictive control

Let the system to be controlled an $\mathrm{ARX}(n_a = 1, n_b = 2, k = 1)$ so that:

$$y(t) = ay(t-1) + b_0u(t-1) + b_1u(t-2) + e(t) \qquad e(t) \sim \mathrm{WN}(0, \lambda^2)$$

Under the standard hypotheses for **Minimum Variance (MV) control**, the MV control law for a reference signal $y^0(t)$ is:

Controlli automatici - 6 CFU

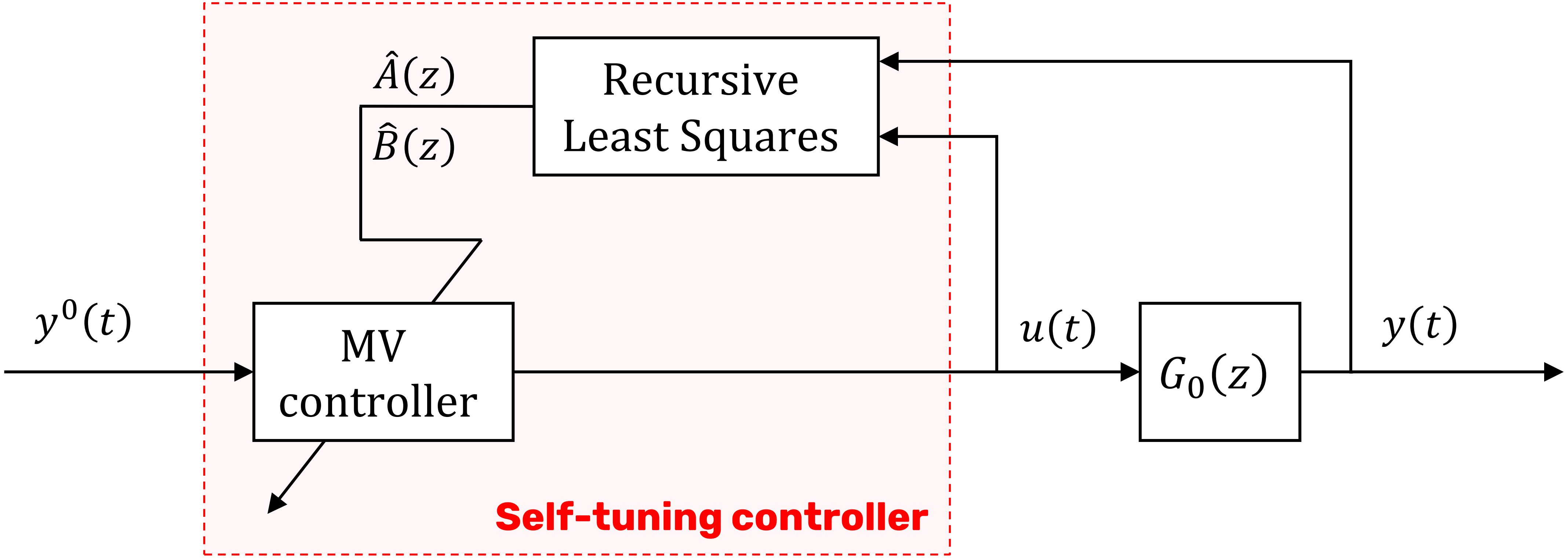
$$u(t) = \frac{1}{b_0}\left(-b_1u(t-1) - ay(t-1) + y^0(t)\right)$$

If the system changes its parameters in time, this control law will be **no more «optimal»** in terms of the **tracking error variance**

$$\mathbb{E}\left[\left(\hat{y}(t|t-1) - y^0(t)\right)^2\right]$$

Example: self-tuning predictive control

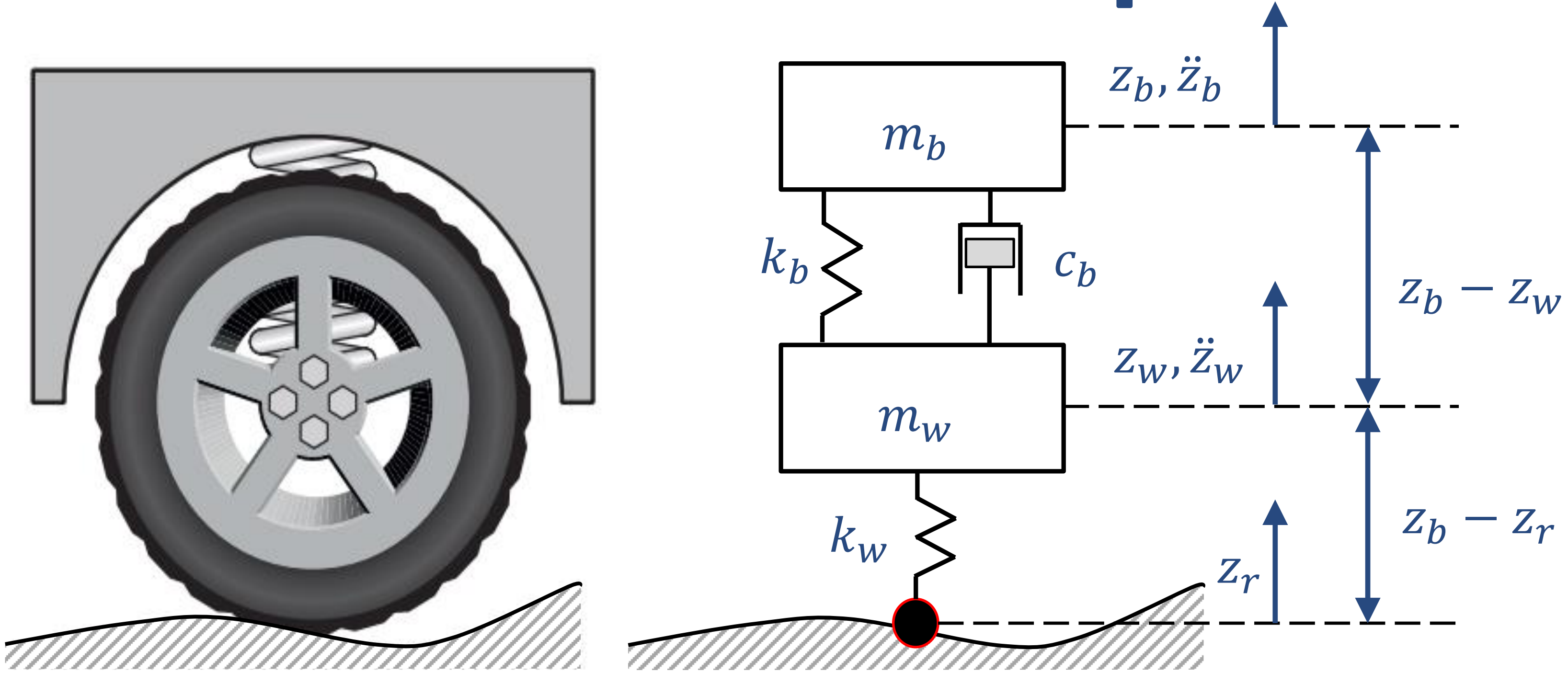
Thus, an **adaptive (self-tuning) control scheme** is envisaged:



The estimation algorithm will provide the estimates $\hat{a}(t), \hat{b}_0(t), \hat{b}_1(t)$ at **each time** t

Example: supervision of an automotive suspension

The **suspension** and the **tire pressure** have a large influence on the vehicle dynamics and are thus highly **safety critical**



Consider a (simplified) **quarter-car** model, where:

- m_b, m_w: body and wheel mass
- $z_b - z_w$: suspension deflection
- k_w: wheel rigidity
- k_b, c_w: suspension rigidity and damping coefficients
- z_r: road level

Example: supervision of an automotive suspension

Force balances lead to:

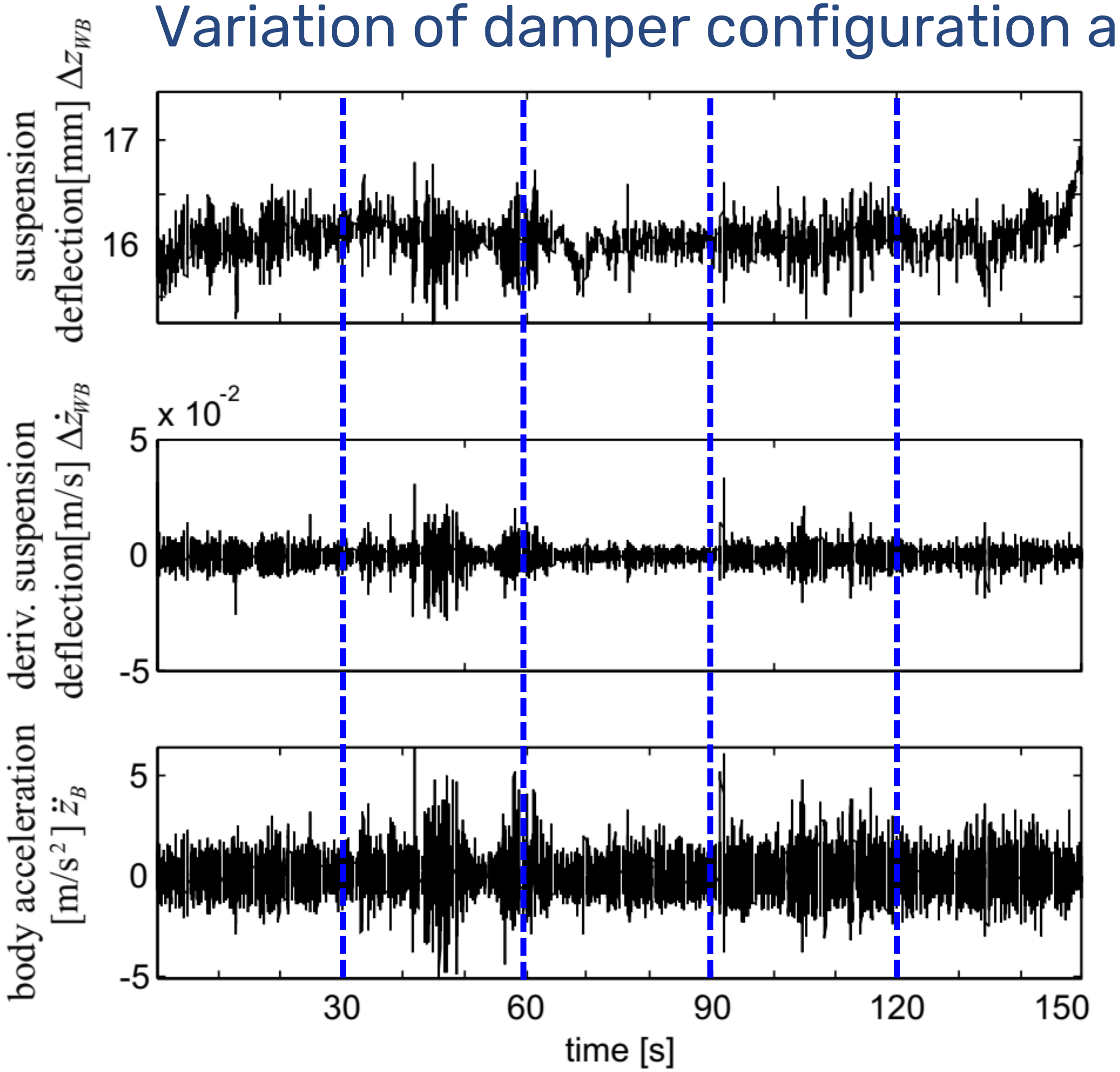
$$\begin{cases} m_b \ddot{z}_b(t) = -k_b\big(z_b(t) - z_w(t)\big) - c_b\big(\dot{z}_b(t) - \dot{z}_w(t)\big) - m_b g \\ m_w \ddot{z}_w(t) = k_b\big(z_b(t) - z_w(t)\big) + c_b\big(\dot{z}_b(t) - \dot{z}_w(t)\big) - k_w\big(z_w(t) - z_r(t)\big) - m_w g \end{cases}$$

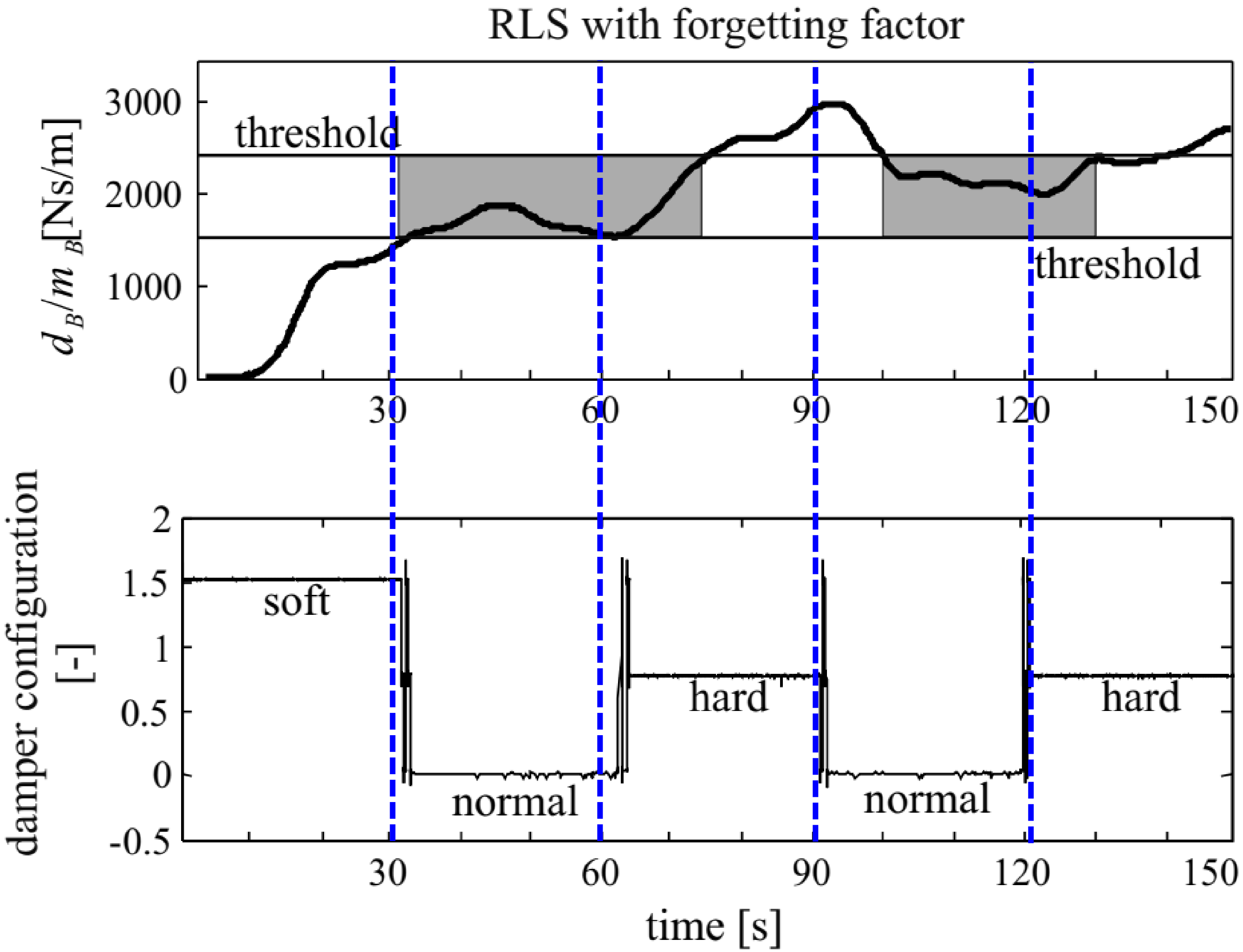
Assume that the body mass m_b and damper spring rigidity k_b are known. It is possible to **estimate the suspension viscous coefficient** c_b with **linear regression** measuring the body acceleration $\ddot{z}_b$ and the suspension deflection $z_b - z_w$

However, for a **fault detection** purpose (e.g. a **worn out of the suspension** which cause a change in the damping coefficient) **recursive methods** are needed

Example: supervision of an automotive suspension

Variation of damper configuration at $t = 30, 60, 90, 120\ s$





Adaptive Learning, Estimation and Supervision of dynamical systems

B) ESTIMATION AND IDENTIFICATION

Until now, we focused on **Prediction Error Methods (PEM)** for **SISO** LTI system identification. These methods are optimal if some assumptions holds:

- The input and output signals are assumed to be **stationary** stochastic processes (ssp), so that **no transients** must be present in the signals
- The input has to **be persistently exciting** of a sufficient order
- Only **input-output models** can be identified

Furthermore, they may optimize a **non-convex** cost function

Adaptive Learning, Estimation and Supervision of dynamical systems

However, sometimes we:

- may have at disposal only signals **with transients**, **not stationary** and **low-order exciting** inputs

- may need a **state-space models** instead of an input-output one (e.g. for Kalman filtering advanced control as **LQR** or **MPC**, model-based **fault diagnosis** approach)

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SUBSPACE IDENTIFICATION METHODS

Adaptive Learning, Estimation and Supervision of dynamical systems

Subspace identification approaches are not based on optimization, and they are often used as an **initialization step** for PEM approaches

However, subspace methods allow a simpler identification scheme for **MIMO systems**, as the **number of parameters** to be estimated is often **lower** with respect to PEM approaches

MIMO SYSTEM IDENTIFICATION

Example: estimate a large MIMO system

Suppose that $m_u = 40$ inputs, $p = 28$ outputs, $n = 22$ states

Let an **Input-Output model** assume that each I/O relation is modeled by a **100° order FIR** model. Then, the **total number of parameters** to be estimated is

$$\text{number of params} = m_u \cdot p \cdot 100 = 40 \cdot 28 \cdot 100 = \boxed{112000}$$

State-space model

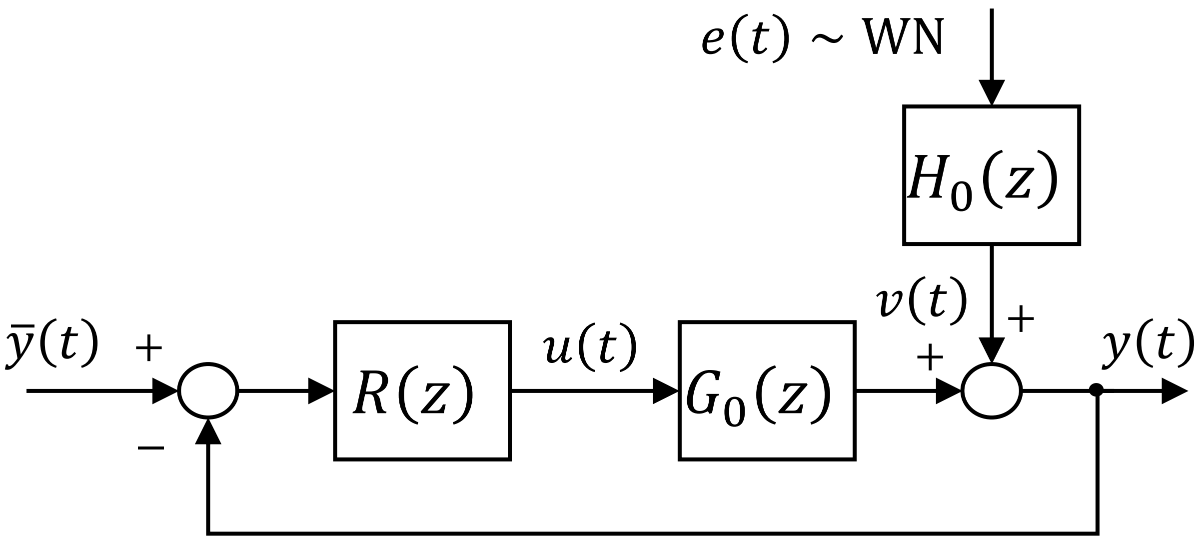
$$\begin{cases} \underset{n \times 1}{\boldsymbol{x}(t+1)} = \underset{n \times n}{A} \cdot \boldsymbol{x}(t) + \underset{n \times m_u}{B} \cdot \underset{m_u \times 1}{\boldsymbol{u}(t)} \\ \underset{p \times 1}{\boldsymbol{y}(t)} = \underset{p \times n}{C} \cdot \boldsymbol{x}(t) + \underset{p \times m_u}{D} \cdot \boldsymbol{u}(t) \end{cases}$$

$$\begin{aligned} \text{number of params} &= n \cdot n + n \cdot m + n \cdot p + p \cdot m \\ &= 22 \cdot 22 + 22 \cdot 40 + 22 \cdot 28 + 28 \cdot 40 \\ &= \boxed{3100} \end{aligned}$$

Adaptive Learning, Estimation and Supervision of dynamical systems

Consider a **closed-loop** SISO control system:

- Usual operation of many plants, in particular if $G_0(z)$ is **open-loop unstable**



- Sometimes the controller is **intrinsically present** (e.g. biomedical, economic systems)

- The controller can have the effect of **«linearizing»** a nonlinear plant

Main (fundamental) difference with the open-loop setting: $u(t)$ and $v(t)$ are **correlated**

Adaptive Learning, Estimation and Supervision of dynamical systems

In most situations the goal of **closed-loop identification** is to estimate $G_0(z)$ and possibly $H_0(z)$. Sometimes, also the estimate of $R(z)$ is of interest. Questions that arise are:

- Are the estimates of $G_0(z)$ and $H_0(z)$ **consistent** when $\mathcal{S} \in \mathcal{M}(\boldsymbol{\theta})$?
- Are the estimates of $G_0(z)$ and $H_0(z)$ **consistent** when only $G_0 \in \mathcal{G}(\boldsymbol{\theta})$?
- Is it possible to **tune the bias** of the estimate?
- What is the **variance** of the estimates?

CLOSED-LOOP IDENTIFICATION METHODS

Adaptive Learning, Estimation and Supervision of dynamical systems

C) SUPERVISION OF DYNAMICAL SYSTEMS

The **supervision (**or **diagnosis)** of dynamical systems aim to **discover anomalies** in the plant, its actuators and sensors, using **measurements** and a **model** of the plant

- How to **detect** if a process component is «healthy» or «faulty»?
- How to **isolate** which component is faulty?
- How to be **robust** despite of *noises*, *disturbances* or *model inaccuracies*?

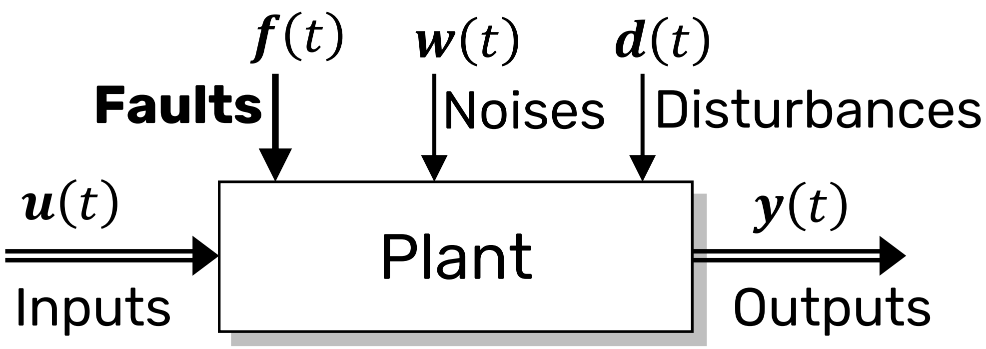
FAULT DIAGNOSIS METHODS

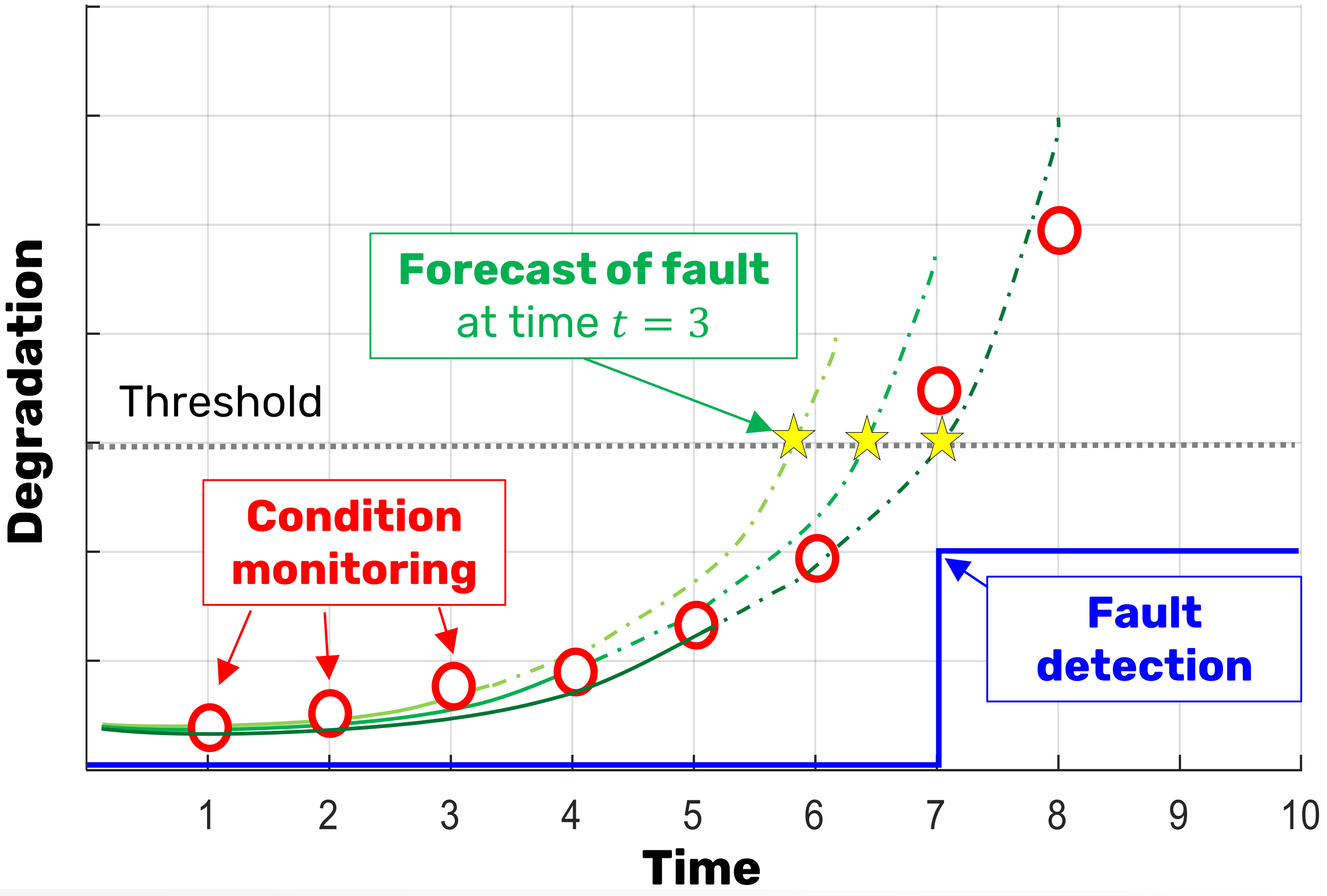
Adaptive Learning, Estimation and Supervision of dynamical systems

1. Fault **detection** and **isolation**

2. Condition **monitoring**

3. **Prognosis**

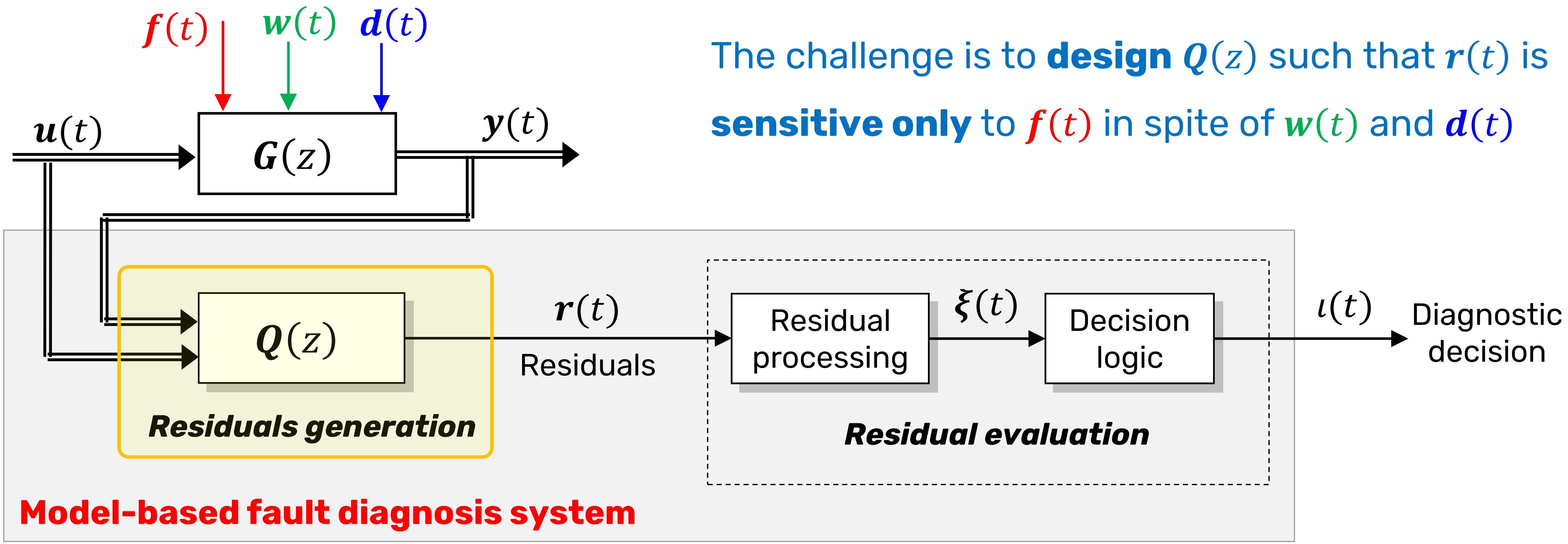




Adaptive Learning, Estimation and Supervision of dynamical systems

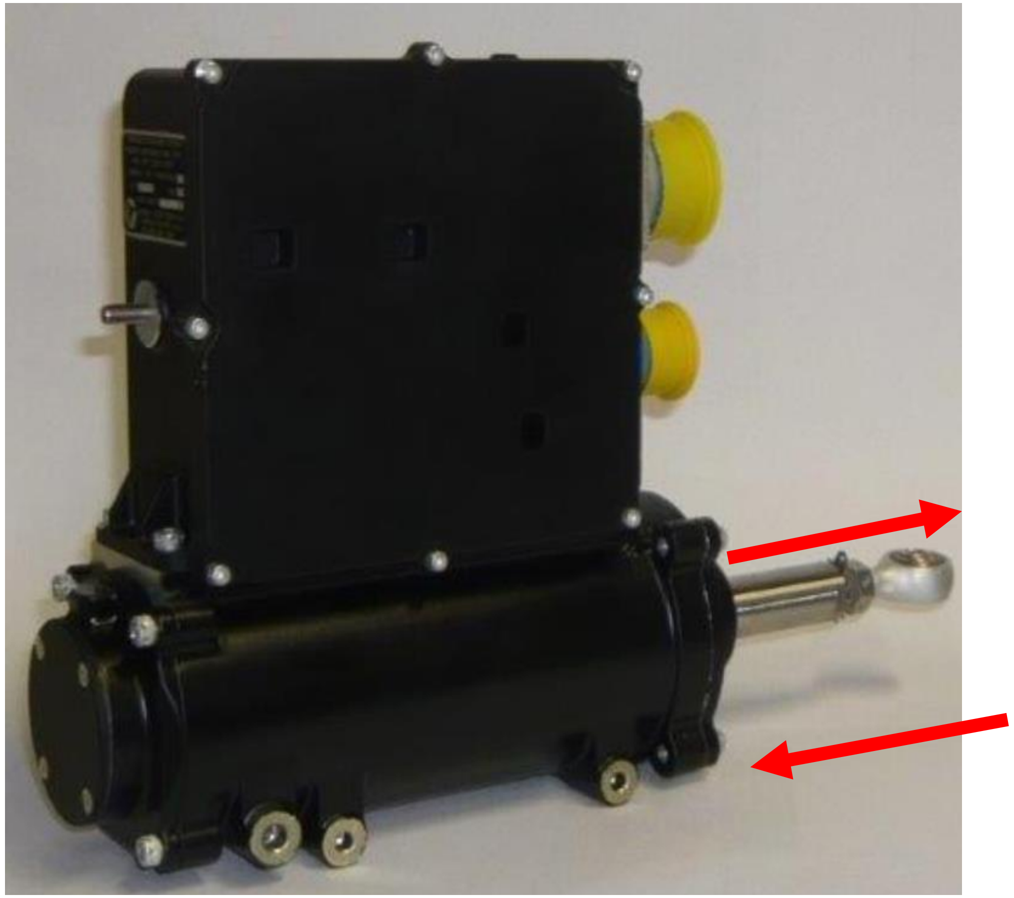
We will mainly focus on the **model-based approach** to fault diagnosis

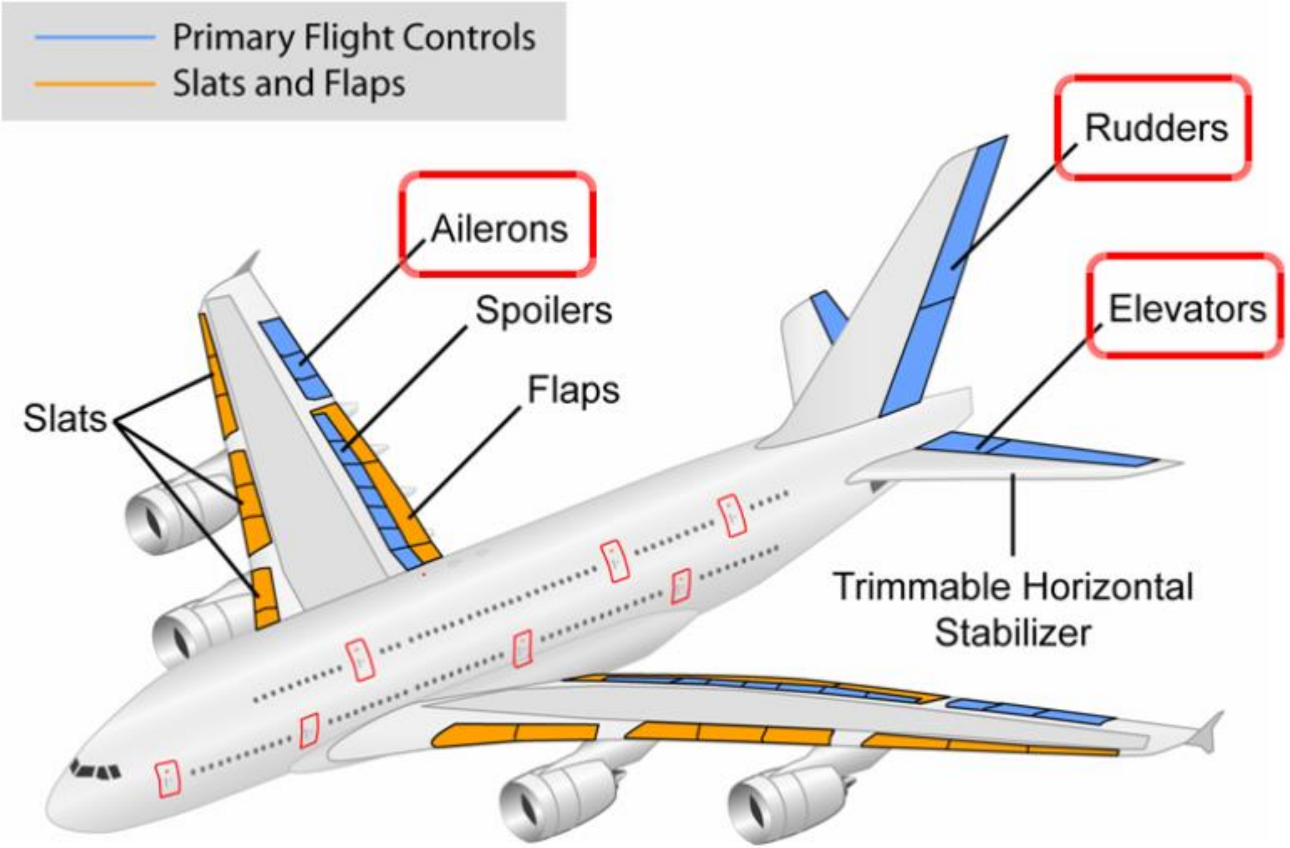
The challenge is to **design** $\boldsymbol{Q}(z)$ such that $\boldsymbol{r}(t)$ is **sensitive only** to $\boldsymbol{f}(t)$ in spite of $\boldsymbol{w}(t)$ and $\boldsymbol{d}(t)$



Example: model-based EMA jamming detection

Consider an EMA actuating **primary flight surfaces:**

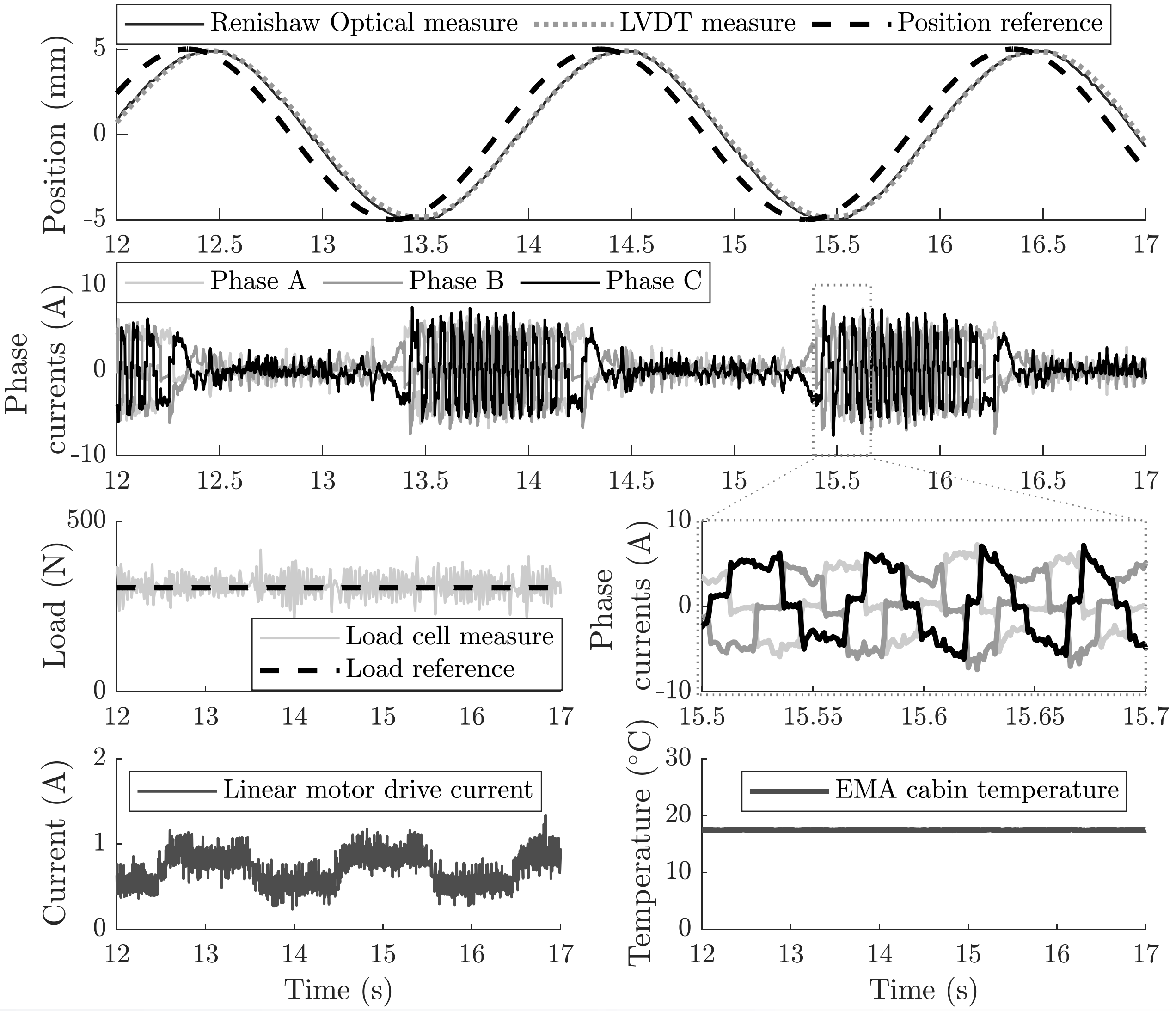




Example: model-based EMA jamming detection

Test bench measurements

1. EMA Phase currents
2. EMA LVDT position
3. Linear motor position
4. EMA Reference position
5. Load cell



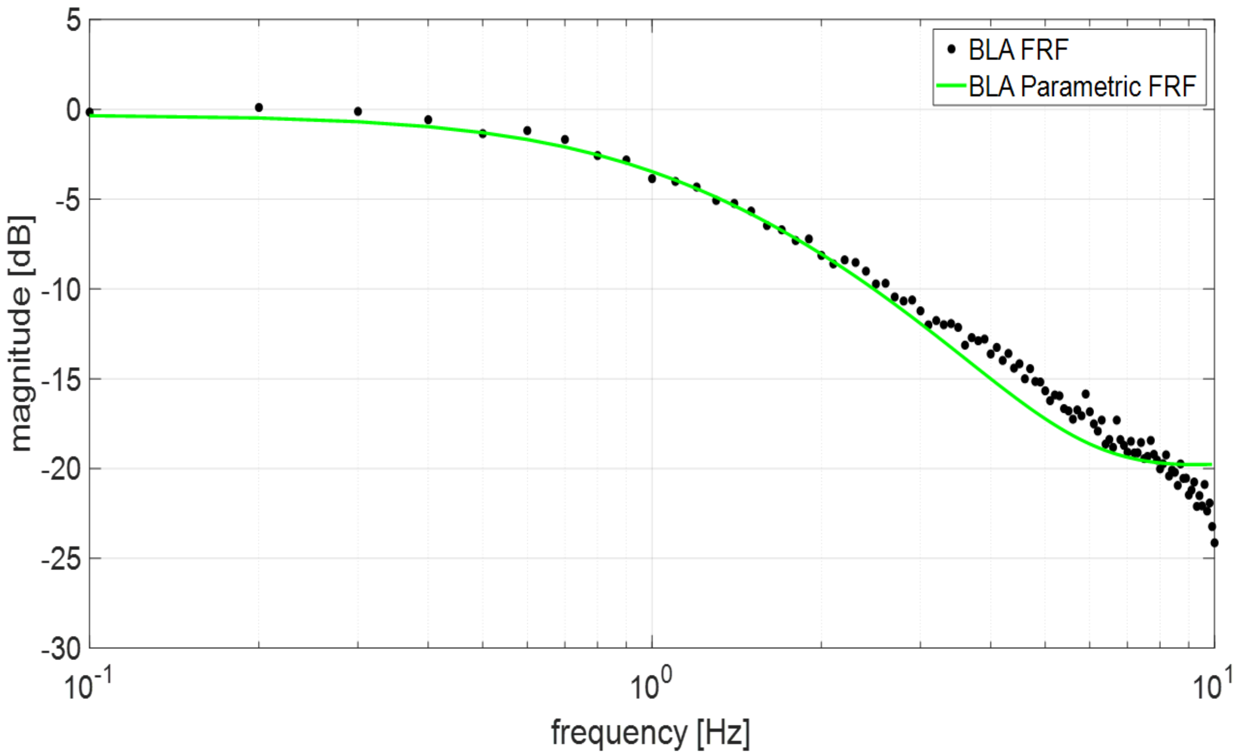
Example: model-based EMA jamming detection

Identification method:

1. **Multisine** excitation

2. **Nonparametric estimate** of the closed-loop transfer function

3. **Parametric estimate** fitting FRF data



$$\frac{X(z)}{\bar{X}(z)} = G(z) = \frac{0.06892 - 0.1732z^{-1} + 0.1266z^{-2}}{1 - 1.624z^{-1} + 0.6467z^{-2}}$$

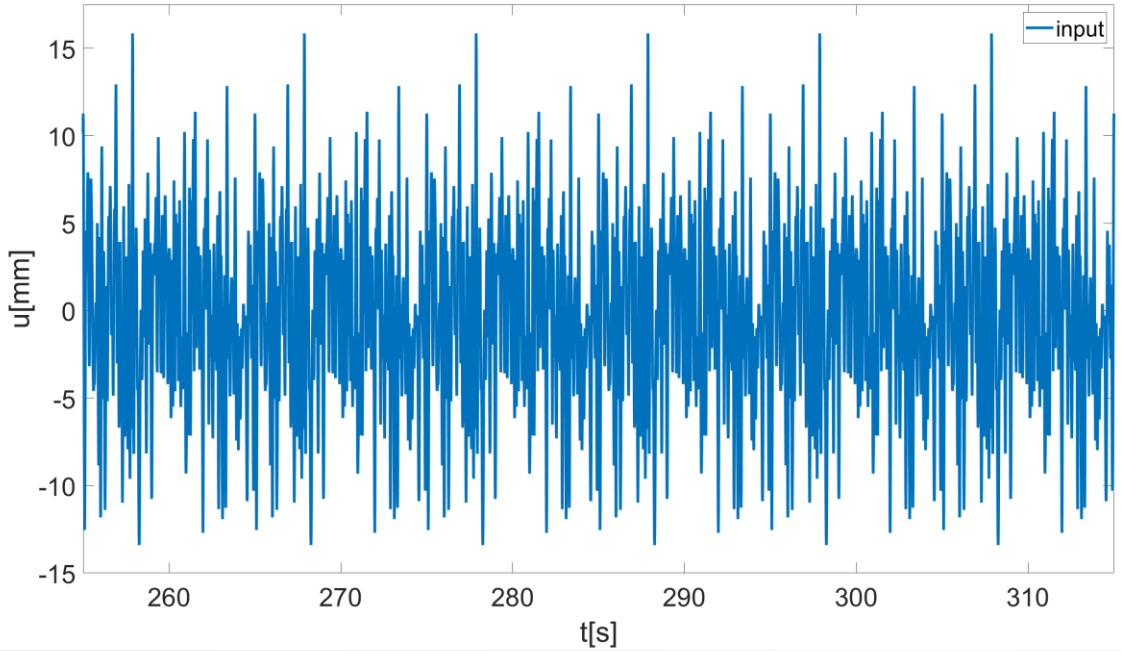
- $\bar{x}(t)$: position reference
- $x(t)$: position measure LVDT

Example: model-based EMA jamming detection

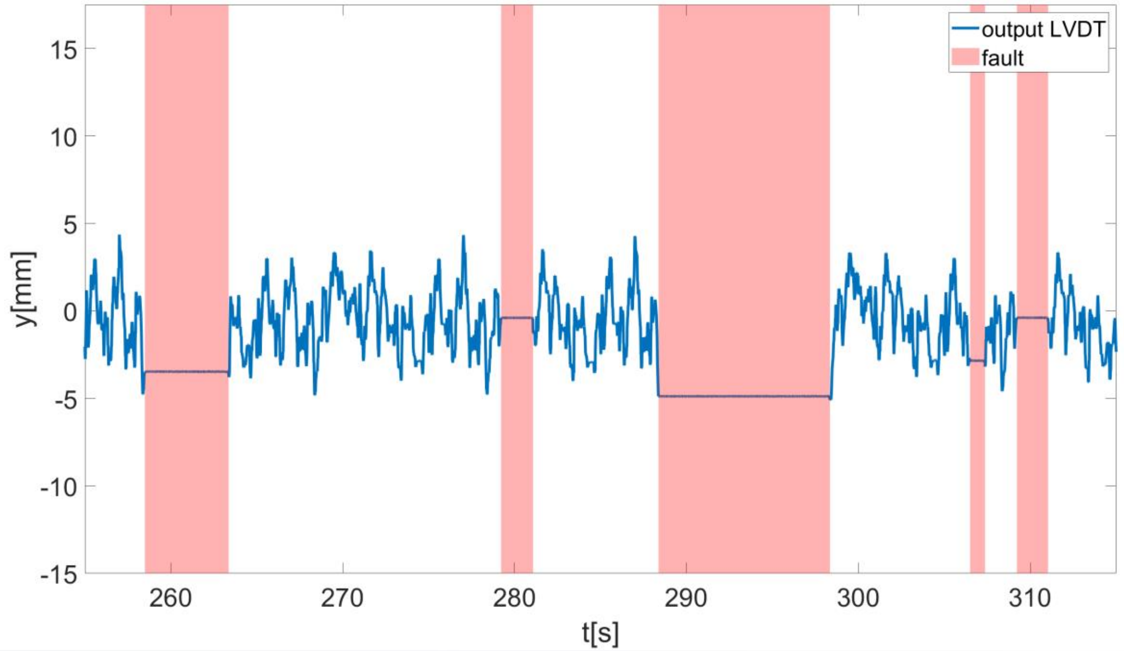
When the degradation took over, the **EMA undergone small-jams** during the operation.

The output position signal, as measured by the LVDT sensor, shows **constant value.** We modeled the fault as an **actuator fault**.

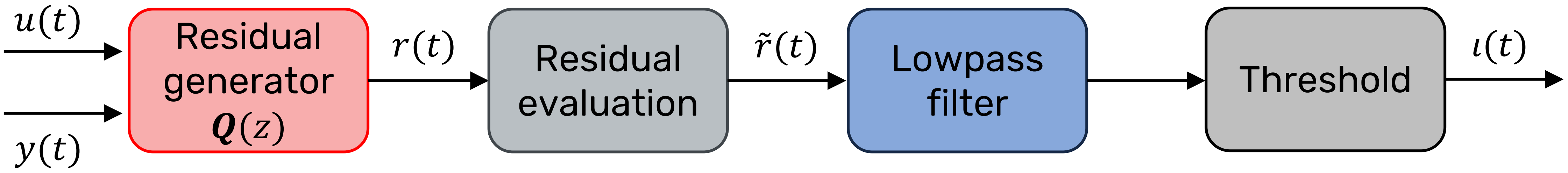
Multisine position reference

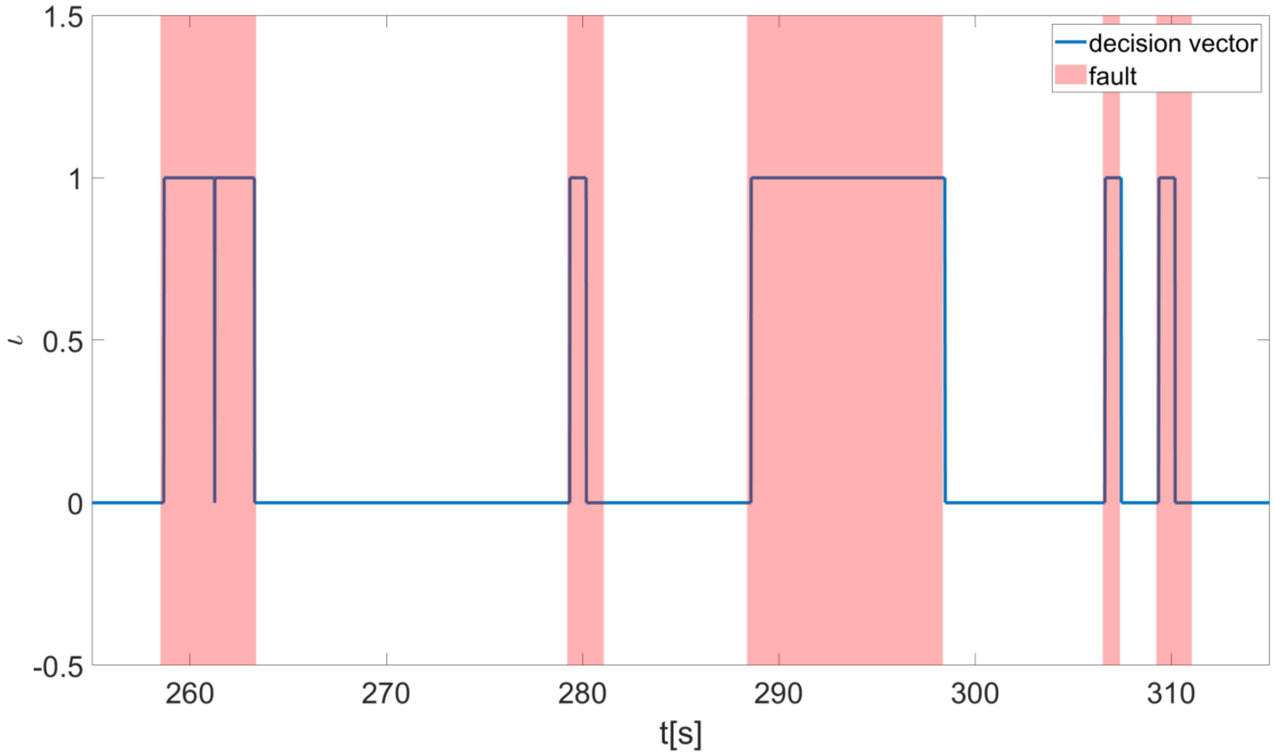


Output position (LVDT)



Example: model-based EMA jamming detection





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